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X-RAY DIFFRACTION IN LIQUIDS: SATURATED NORMAL FATTY ACIDS, ISOMERS OF PRIMARY NORMAL ALCOHOLS, AND NORMAL PARAFFINS

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The following results have been obtained in continuation of the investigation of Stewart and Morrow[1] on X-ray diffraction in liquids. The apparatus and method were the same as before. The conclusions will be presented with brevity anticipating publication in full in the near future.

I. Normal Fatty Acids.—The results for the fatty acids were obtained by Dr. Roger M. Morrow, using liquid samples at room temperature, including the eleven from formic (CH_2O_2) to undecylic ($C_{11}H_{22}O_2$) acids. The results are similar to those with the primary *n*-alcohols in two respects. (1) By the application of Bragg's law, two spacings are found, one of which is the lateral separation of the molecules lying in parallel collinear chains, and the other the separation of the planes passing transversely through the chains and containing the diffraction centers of the molecules. The lateral spacing is 4.55 A.U. for the samples having more than five carbon atoms to the molecule. This is in agreement with the work of Adam and others[2] on monomolecular films. (2) The longitudinal spacing varies linearly with the carbon content, the increase being 2.00 A.U. for two carbon atoms, one for each of two molecules.

As judged by density considerations, the chains are not normal to the transverse planes and in each spacing of the second kind there are two molecules in series, with associated COOH groups. A comparison of spacings in the liquid and crystalline states shows that the arrangement of molecules in liquids, or cybotaxis, is not that found in the crystalline state.

II. Isomers of Primary n-Alcohols.—Nine isomers were investigated and compared with the corresponding primary normal alcohols, one of *n*-butyl, five of *n*-amyl, one of *n*-hexyl and two of *n*-heptyl alcohol. The study was made by Professor E. W. Skinner of Olivet College and the present author. The differences found in the diffraction intensity curves

of an alcohol and its isomer are distinct and several conclusions, quantitative and qualitative, can be drawn. The attachment of CH_3 as a side branch alters the "diameter" by 0.6 A.U., and of OH by 0.4 A.U. The attachment of CH_3 and OH to the same atom in the chain increases the diameter by only 0.65 A.U. The measured "diameter" of the molecule is a mean value, but in the case of di-*n*-propyl carbinol there is a better resolution and the side attachment of OH increases the diameter in one direction by 0.45 A.U. and in the other not at all.

The existence is shown of associated polar groups giving two molecules in a continuous chain for each longitudinal spacing. When the OH group is not attached to the last or next to the last atom in the chain, this association disappears and there is but one molecule for each longitudinal spacing. This is in agreement with the experiments of others on similar chains in the solid state.

In one case, triethyl carbinol, the first, third and fifth orders of the side spacing are apparently present.

III. N-Paraffins.—Pentane, hexane, heptane, octane, nonane, decane, dodecane, tetradecane and pentadecane were studied by the author. There are five experimental points of interest.

1. In the ionization diffraction curve there is one peak only, instead of two as with the alcohols and fatty acids. The absence of the second peak, which would indicate a longitudinal spacing, is explained qualitatively by a consideration of the longitudinal molecular forces. These could not produce as uniform a longitudinal arrangement as occurs in the other compounds.

2. The single peak is located at the same angle with each *n*-paraffin, excepting pentane and decane. These two are believed to be isomers and efforts will be made to secure and investigate the normal compounds. The lateral separation of molecules is approximately 4.6 A.U. and is essentially the same as in the liquids previously studied and referred to in this paper.

3. In six of the nine cases the diffraction intensity does not increase as one approaches 0°. The significance is not clear, though one is reminded of total reflection.

4. If one assumes the C atom to occupy a length in the chain of 1.3 A.U., as found by other observers of X-ray diffraction in solids, and the length of H, 1.0 A.U., and if one assumes the volume occupied by a molecule to be the square of the lateral spacing times the molecular length, then the computed values of densities of the seven normal paraffins are correct to within less than 4% or a mean of 2%.

5. Two additional but faint peaks in the diffraction intensity curves are found with pentadecane and tetradecane. The corresponding spacings are 2.1 A.U., and 1.23 A.U. The nearness of these to the repetitive values of 2.0 A.U. and 1.26 A.U., which are found in a diamond, is suggested.

All of the foregoing indicates the importance of the X-ray diffraction study of liquids and adds strong evidence in favor of the interpretation on the basis of a molecular space array. The existence of this array or cybotaxis is of fundamental interest in an understanding of physical phenomena in liquids.

[1] Stewart and Morrow, *Physic. Rev.*, **30**, 9, 232 (1927).
[2] Adam, *Proc. Roy. Soc.*, **A101**, p. 452 (1922).

THE STRUCTURE AND DIMENSIONS OF THE BENZENE RING

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In a previous paper[2] the spherical atoms proposed by W. L. Bragg and W. P. Davey have been specialized and the cubic atom proposed by Lewis and developed by Langmuir extended to simple polyhedrons inscribable in spheres. It was shown in detail how these simple models could be built up into the various types of observed cubic and hexagonal lattices and how the lattice constants were geometrically related to the atomic radii. In the case of diamond and of graphite, it was found that these lattices could be built up of cubes having one corner (electron) shared in common and that the radius (half the body diagonal) was equal to 0.77 Å in diamond and 0.75 Å in graphite, giving a mean value for the radius of the carbon atom of 0.76 Å over the extremely wide range of physical conditions between diamond and graphite.

This paper is an account of the application of these ideas to the well-known problem of the structure of the benzene ring. A model of the benzene ring built up of cubes having corners shared in common, will first be presented by giving the coördinates of the cube centers (carbon nuclei) and the cube corners (possible electron positions) in terms of a parameter R (radius of the carbon atom) as is customary in point group theory. This will be followed by a note on the construction of a model of the benzene ring at a scale of two inches to the Ångstrom which represents a magnification of approximately 500 million diameters. After this there will be a brief discussion of the properties and possibilities of this model.

Coördinates of the Benzene Ring in Terms of R, the Radius of the Carbon Atom.—Cube Centers, Carbon Nuclei.

$$\pm 1.138, \quad 0.000, \quad \pm 0.804$$

$\pm 0.569, \mp 0.986, \mp 0.804$
$\pm 0.569, \pm 0.986, \mp 0.804$

Cube Corners, Possible Electron Positions.

$-1.000, \pm 0.577, \quad 0.000$
$+1.000, \pm 0.577, \quad 0.000$
$\quad 0.000, \pm 1.155, \quad 0.000$
$\mp 0.667, \quad 0.000, \pm 0.943$
$+0.333, \pm 0.577, +0.943$
$-0.333, \pm 0.577, -0.943$
$+1.276, \pm 0.577, +1.609$
$-1.276, \pm 0.577, -1.609$
$+1.943, \pm 0.577, +0.665$
$-1.943, \pm 0.577, -0.665$
$-0.152, \pm 1.392, +1.609$
$+0.152, \pm 1.392, -1.609$
$+0.475, \pm 1.971, -0.665$
$-0.475, \pm 1.971, +0.665$
$+1.474, \pm 1.392, -0.665$
$-1.474, \pm 1.392, +0.665$
$-1.138, \pm 0.817, +1.609$
$+1.138, \pm 0.817, -1.609$

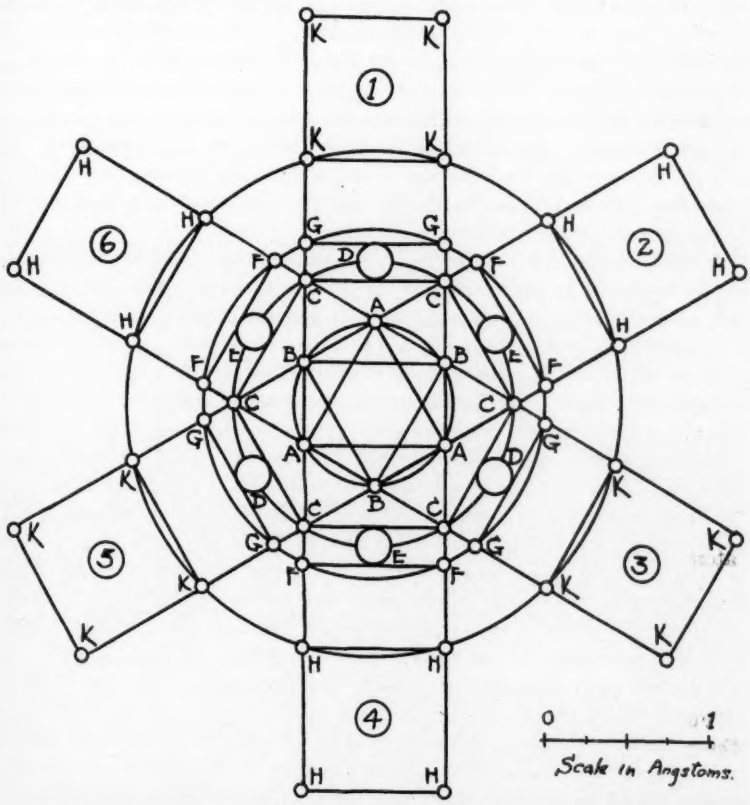
A study of these coördinates indicates that twelve cube corners (electron positions) are on the surface of a sphere whose radius is 1.155 R, that the six cube centers (carbon nuclei) are located on the surface of a concentric sphere of radius 1.394 R and that the remaining twenty-four cube corners (possible electron positions) are all on the surface of a third concentric sphere whose radius is 2.134 R.

The Construction of a Scale Model of the Benzene Ring.—For the study of the properties of complex lattice and molecular structures three dimensional models are a necessity. It is also convenient to have them made to scale. For a study of the general arrangement of the atoms a scale of 2.5 cm. to the Ångstrom is most convenient, but for studying the fine structure it is better to double this and use 5.0 cm. to each 1×10^{-8} cm. An inexpensive and simple way of making these models is to lay off a plan, at the scale required, on a wooden base and represent the nuclei and "possible electron positions" by colored spheres set on vertical rods.

Figure 1 is a scale plan of this model of the benzene ring together with the approximate locations of the atoms of the possible substituted groups attached to the ring which are lettered (1), (2), (3), (4), (5) and (6).

To construct a model at a scale of five centimeters to the Ångstrom: The radii of the circles through the points *C*, *F* and *H* are, approximately,

4.4 cm., 5.3 cm. and 6.2 cm., respectively. The vertical positions measured in centimeters above (+) and below (−) the equatorial plane of the model are, Centers of carbon atoms (*B*) +3.1 cm., (*E*) −3.1 cm., Centers of substituted atoms (1), (3) and (5) −0.3 cm., (2), (4) and (6) +0.3 cm., Cube corners (possible electron positions) (*A*) +3.6 cm., (*B*) −3.6 cm., (*C*) 0.0 cm., (*F*) +6.1 cm., (*G*) −6.1 cm., (*H*) +2.5 cm. and −1.9 cm., (*K*) −2.5 cm. and +1.9 cm.



These dimensions are only approximate and more accurate values can easily be found by multiplying the last column in the preceding table by 5 × 0.76. The result will be the number of centimeters required.

Discussion.—The three salient requirements which must be fulfilled by any proposed structure for the benzene ring are most beautifully illustrated by the scale model herewith presented. (1) *Isomerism.* Since the atoms of the substituted groups united to the carbon atoms of the ring form in

the model an hexagonal arrangement lying very nearly in a plane, all the observed isomerism of the substituted derivatives of benzene can be simply and beautifully illustrated. (2) *Unsaturation.* The unsaturated character of the benzene is represented on the model by the six vacant "electron positions" lying on the outer sphere. A study of the model shows that there are twenty-four "possible electron positions" on this surface and only eighteen electrons are available to fill them. The addition of six hydrogen atoms will completely fill all the "possible electron positions" and the ring will be saturated. The formation of hydro-benzene derivatives and addition compounds are thus illustrated by this model. (3) *Optical Activity.* A study of this model shows that each carbon atom of the ring is attached not to free carbon atoms but to a four-atom ring of carbon atoms. Hence no matter what groups are attached there will not be an "asymmetric" carbon atom in the ring forming this model. Therefore, this model illustrates the observed non-occurrence of optically active isomers among benzene derivatives.

It will be noted that nothing has been said so far in regard to the position of the hydrogen atoms in benzene itself or in its substituted compounds. The modern notion of the hydrogen atom in combination is that the single electron which belongs to it fills in one of the "electron positions" of the atom attached leaving the hydrogen atom alone in the space outside of the attached atom. The hydrogen nucleus, even at the enormous magnification of these models (and the same applies to all other nuclei), can be considered simply as a point without volume. It is only in rare cases, such as lithium hydride, where negative hydrogen is known to exist that the hydrogen atom in combination can be thought of as occupying an appreciable volume. With all other atoms this is not the case and their volume must be taken into account. *Hence the positions in space of the hydrogen atoms attached to carbon atoms may well be quite different from the positions of the atoms of the substituted groups attached to carbon atoms which have been substituted for hydrogen.* This possibility may account for the well-known inert character of benzene itself compared with the relative ease with which chemical reactions take place when one of the hydrogen atoms has been removed and a group such as OH or CH_3 has been added in its place. For the solutions of the problem of the position of the hydrogen nuclei in benzene itself, the application of the model herewith presented to the interpretation of the X-ray data on crystalline benzene seems a very promising lead.

Furthermore, this model may offer an important clue to the solution of the problem of determining whether a given benzene mono-derivative shall give a meta-di-derivative or a mixture of ortho- and para-di-derivatives. The explanation of the so-called Crum-Brown Rule and its deviations is undoubtedly bound up in the problem of unsaturation mentioned

above. For example, does each carbon atom of the ring in phenol lack one electron of filling its eight "possible electron positions" or do three of the carbon atoms lack two electrons each while the remaining three carbon atoms have all their "possible electron positions" filled, or is there some other combination?

In conclusion, the main purpose of presenting this detailed scale model of the benzene ring at this time is to offer a means by which the results of the X-ray analysis of benzene derivatives can be interpreted and the fine structure of their lattices determined.

Finally, I wish to express my thanks to Professor Henry G. Gale for his interest in this work.

[1] This investigation was commenced by the author as a NATIONAL RESEARCH FELLOW.

[2] J. K. Morse, *Proc. Nat. Acad. Sci.*, **13**, 227–232, 1927.

THE SPACE GROUP OF POTASSIUM, RUBIDIUM AND CAESIUM SULPHATES

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While, recently, a considerable number of papers[1] have appeared detailing investigations on the anhydrous sulphates of type RSO_4, very little attention has been given to the study of anhydrous sulphates of type R_2SO_4.

In 1916, Ogg and Hopwood[2] made a preliminary investigation of the structure of the sulphates of potassium, rubidium, caesium and ammonium sulphates by means of reflection measurements and suggested that an elementary orthorhombic lattice containing four molecules seemed in best accord with their data but they did not determine the space group nor the fine structure of these sulphates.

Bradley,[3] using the powder method of analysis, examined the crystal structure of lithium potassium sulphate and while his analysis of the part which the structure of the SO_4 ion plays in determining the nature and dimensions of the crystal lattice is of interest it does not throw much light on the internal arrangement of the large number of orthorhombic salts of the type R_2SO_4 which crystallize without water of crystallization.

Vegard,[4] in studying the constitution of mixed crystals and the packing effect of the atom, made use of the powder method of analysis as developed by Debye and Scherrer,[5] and independently by Hull.[6] He includes among his photographs one of potassium sulphate and one of ammonium sulphate but gives no further data except to place them in the orthorhombic system and to class them as isomorphous.

In order to find the crystal structure of the orthorhombic salts of the type R_2SO_4 and to determine, if possible, the space group of the sulphates of potassium, rubidium and caesium an investigation by the method of Laue spots was undertaken. The results of this investigation comprise this paper.

Crystallographic Measurements and Laue Photographs.—Goniometric measurements by Tutton[7] show that potassium, rubidium and caesium sulphates are isomorphous and belong to the Rhombic Bipyramidal or Holohedral class.[8]

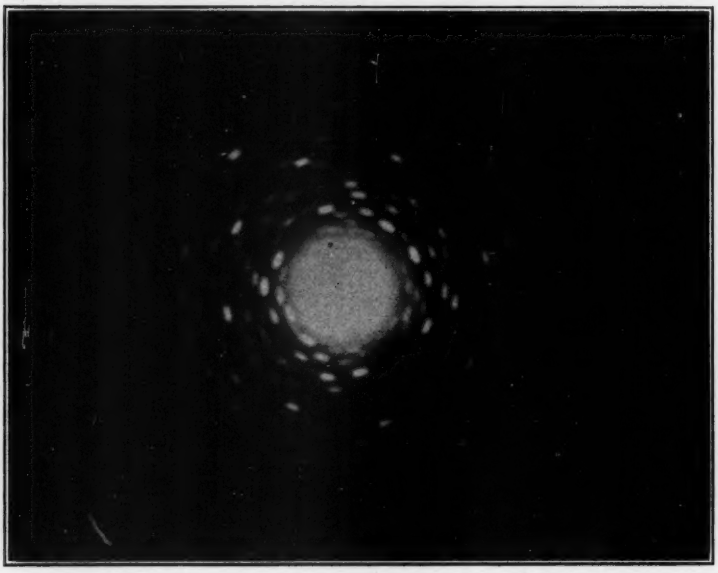

FIGURE 1

Laue photograph of a crystal of potassium sulphate X-rays normal to the 010 face.

The axial ratios and densities determined by Tutton are given below:

K_2SO_4	$a:b:c = 0.5727:1:0.7418$	Density 2.666 gm./cm.^{3}
Rb_2SO_4	$a:b:c = 0.5723:1:0.7485$	Density 3.615 gm./cm.^{3}
Cs_2SO_4	$a:b:c = 0.5712:1:0.7531$	Density 4.246 gm./cm.^{3}

The crystals used in this investigation were grown by slow evaporation from cold, concentrated solutions. Whenever it was desired to hasten the growth the solutions were evaporated over concentrated sulphuric acid. The most perfect crystals were selected, mounted and cut into sections parallel to the 100, 010 and 001 faces and cut to a thickness of 0.5 mm.

From these, Laue photographs were obtained by use of a Coolidge tube with a tungsten anticathode in the usual manner. Figure 1 shows a typical plate of the fourteen used. Gnomonic projections[9] were then made, the indices of the planes determined and the values of $n\lambda$ computed. Figure 2 shows one of these projections.

Determination of the Space Lattice.—Lattice constants for the sulphates

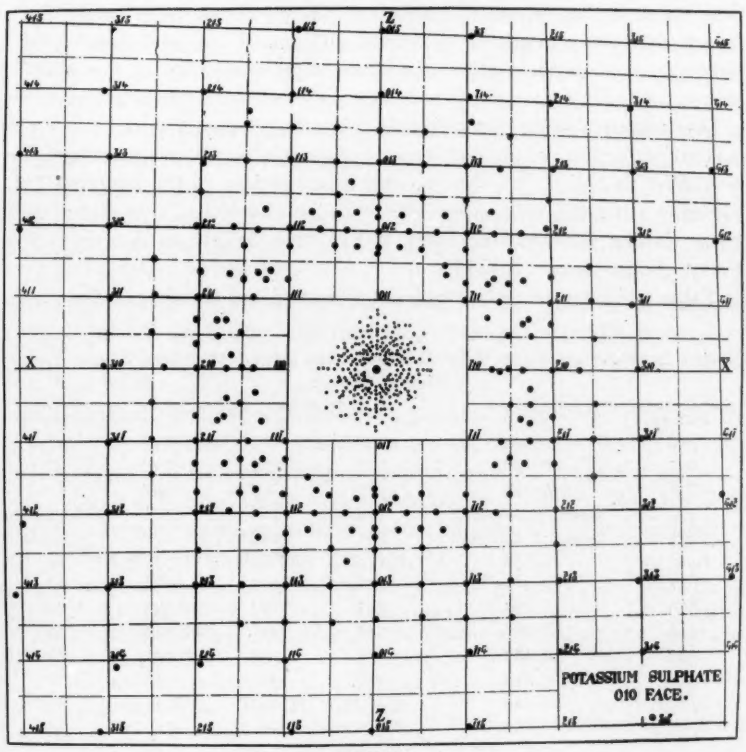

FIGURE 2

used in this study were determined by Ogg and Hopwood[2] and are as follows:

K_2SO_4 ... $a_0 = 5.746$ A.U. ... $b_0 = 10.033$ A.U. ... $c_0 = 7.443$ A.U.
Rb_2SO_4 ... $a_0 = 5.949$ ... $b_0 = 10.394$ ... $c_0 = 7.780$
Cs_2SO_4 ... $a_0 = 6.218$ ... $b_0 = 10.844$ ... $c_0 = 8.198$

The voltages across the tube were measured by calibrated sphere gaps and at no time during the exposure did the root mean square voltage exceed 56,600 volts and for the majority of the exposures the tube operated

at 50,000. Hence the minimum wave-length emitted was found to be 0.218 Ångstrom from the well-known relation,

$$h\nu = Ve$$

The values of $n\lambda$ for each plane producing spots were computed from the photographs by means of Bragg's relation modified to suit a simple orthorhombic lattice and all observed values of $n\lambda$ were found to be larger than the computed minimum. Hence it was concluded that a simple orthorhombic lattice containing four molecules is the smallest elementary unit lattice which is in agreement with the diffraction patterns.

Determination of the Space Group.—Since the elementary unit is a simple orthorhombic lattice, the only space groups fulfilling both these conditions are $2Di$-1 to $2Di$-16,[10] inclusive. By a comparison of the observed Laue patterns with the diffraction effects to be theoretically expected from these sixteen possible structures, it has been found possible to exclude every group except $2Di$-13.

Table 1 gives a list of the typical types of planes showing on the photo-

TABLE 1

X-RAY DIFFRACTION DATA UPON K_2SO_4 PLANES WITH ONE INDEX ZERO. PLANES WHICH APPEAR

CRYSTALLOGRAPHIC INDICES	$n\lambda$	LEFT-HANDED INDICES IF $b = x$	$a = x$	$c = x$	INTENSITY OBSERVED
015	0.452	105	051	510	60
027	0.450	207	072	720	40
730	0.378	370	703	037	80
110	Spect. 2, 3 orders	110	101	011	50, 100
021	Spect. 1, 2 orders	201	012	120	40, 100
0, 6, 13	0.359	6, 0, 13	0, 13, 6	13, 6, 0	40
037	0.427	307	073	730	50
$\bar{5}20$	0.470	250	502	025	20
410	0.460	140	401	014	30
190	0.463	910	109	091	50
510	0.310	150	501	015	20
3, 0, 13	0.340	0, 3, 13	3, 13, 0	13, 0, 3	15
5, 0, $\bar{1}9$	0.240	0, 5, 19	5, 19, 0	19, 0, 5	15

TABLE 2

X-RAY DIFFRACTION DATA UPON K_2SO_4 PLANES WITH ONE INDEX ZERO. PLANES ABSENT BUT IN POSITION TO REFLECT

CRYSTALLOGRAPHIC INDICES	$n\lambda$	LEFT-HANDED INDICES IF $b = x$	$a = x$	$c = x$
$\bar{7}20$	0.273	270	702	027
$\bar{9}40$	0.317	490	904	049
$02\bar{9}$	0.262	209	092	920
$03\bar{8}$	0.477	308	083	830
$02\bar{7}$	0.411	207	072	720
$20\bar{9}$	0.438	029	290	902
2, 0, $\bar{1}1$	0.292	0, 2, 11	2, 11, 0	11, 0, 2
3, 0, 14	0.285	0, 3, 14	3, 14, 0	14, 0, 3

graphs for potassium sulphate, and the photographs for rubidium and caesium sulphates were found to be quite similar. Table 2 presents similar data upon absent planes.

In the geometrical theory of crystal structure it has been customary to use the left-handed axes rather than the more rational right-handed axes used in goniometric measurements. This condition necessitates an inversion of the a and b indices as shown in column three.

The correct choice between the various possible space groups in the orthorhombic holohedral class of symmetry is made by a comparison of the indices when $b = x$, $a = x$ and $c = x$, respectively, for the general positions of the sixteen space groups. It is invariably true[10] that reflections which are absent from the general positions of a space group will be missing also in all their special positions. The application of the above criteria placed each of the three sulphates in space group $2Di$-13. Potassium and rubidium sulphates have space group $2Di$-13 when $a = x$ and caesium sulphate when $b = x$.

Conclusion.—From a comprehensive and critical analysis of the Laue photographs of potassium, rubidium and caesium sulphates, it has been found that the experimental data agrees only with space group $2Di$-13, (V_h^{13}) for the internal symmetry of these salts.

The internal symmetry of these compounds having been uniquely determined, it is possible to proceed with the determination of the fine structure of these salts which will be considered in a later investigation.

In conclusion, the writer takes pleasure in thanking Mr. Jared K. Morse, who suggested the problem, for his interest and helpful suggestions throughout this investigation.

[1] R. W. James and W. A. Wood, *Proc. Roy. Soc., Lon.*, **109A**, 598, 1925. Linus Pauling and P. H. Emmett, *J. Amer. Chem. Soc.*, **47**, 1026, 1925. W. Basche and E. Pohland, *Zeit. Electro-Chem.*, **31**, 523, 1925. W. Basche and H. Mark, *Zeit. Krist.*, **64**, 1, 1926.

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[3] A. J. Bradley, *Phil. Mag.*, **49**, 1225, 1925.

[4] Von L. Vegard, *Zeit. Physik*, **5**, 17, 1921.

[5] P. Debye and P. Scherrer, *Phys. Zeit.*, **17**, 277, July, 1916.

[6] A. W. Hull, *Phys. Rev.*, **10**, 661, 1917.

[7] Alfred E. Tutton, *Trans. Chem. Soc., Lon.*, **65**, 632, 1894.

[8] P. Groth, *Chemische Krist.*, **2**, 337–44, 1908, Leipzig.

[9] R. W. G. Wyckoff, *Am. J. Sci.*, **50**, 323, 1920.

[10] *Ibid.*, **59**, 156, 1925.

THE DIAMAGNETIC SUSCEPTIBILITY OF HYDROGEN MOLECULE AND OF HELIUM IN THE NEW QUANTUM MECHANICS

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In a paper by Van Vleck that appeared in these PROCEEDINGS,[1] a formula for the diamagnetic susceptibility of atoms and molecules in the new quantum mechanics was found which replaces Pauli's formula in the classical quantum theory. It gives the molar diamagnetic susceptibility

$$X_M = -(e^2L/6\mu c^2)\sum\overline{r^2} \tag{1}$$

where $\overline{r^2}$ is the time average of r^2 for one electron, or a diagonal element in the matrix for r^2; the summation extends over all electrons in an atom or a molecule.

In a recent paper by the author,[2] the hydrogen molecule problem was investigated by following the line of the Ritz method of solving variational problems. The function finally decided upon to approximate the solution of the wave equation for the normal hydrogen molecule is of the form

$$\psi = A\left[e^{-\frac{1.166(r_1+p_2)}{a}} + e^{-\frac{1.166(r_2+p_1)}{a}}\right] \tag{2}$$

where a = radius of 1_1 orbit, r_1 and p_1 are the distances of one of the electrons to the two nuclei, r_2 and p_2 those of the other electron and A is the constant of normalization. It was also found that when the potential energy of the molecule is a minimum the separation between the nuclei is $D = 1.406a$. In fact this equilibrium configuration is identified as the normal hydrogen molecule.

With these data given, we proceed to evaluate X_M for the molecule. The quantity r^2 in Van Vleck's formula is here evidently the square of the median from one of the electrons to the mid-point between the nuclei. Calling these two squares M_1^2 and M_2^2 for the two electrons, we have

$$\sum\overline{r^2} = \int (M_1^2 + M_2^2)\psi^2\, dV_1\, dV_2 \tag{3}$$

where $dV_1 = r_1^2 \sin\theta_1\, dr_1\, d\vartheta_1\, d\varphi_1$ and similar expressions for dV_2. Besides, we have the relations

$$M_1^2 = (r_1^2 + p_1^2)/2 - D^2/4, \qquad M_2^2 = (r_2^2 + p_2^2)/2 - D^2/4. \tag{4}$$

After we introduce (2) and (4) into (3), the latter integrates explicitly into

$$\sum\bar{r}^2 = \frac{2A^2\pi^2a^2}{(2 \times 1.166)^8}\,[1536 + 64Q^2 + 4e^{-Q}(Q^2 + 6Q + 12)(Q^5/5 + 2Q^4 + 12Q^3 + 48Q^2 + 144Q + 288)/3Q] - D^2/2$$

where $Q = 1.166 \times 2D$. For $D = 1.406a$, we get

$$\sum\bar{r}^2 = 5.90\ a^2 \quad \text{and} \quad X_M = -4.71 \times 10^{-6}.$$

Similar calculations of X_M have been made for the helium atom by utilizing the results of Kellner[3] who computed the energy value of helium according to the new quantum mechanics also by the Ritz method. The final approximative function has the form

$$\psi = e^{-\frac{\eta_1+\eta_2}{2}}\,[C_1L_1'(\eta_1)L_1'(\eta_2)P_0(x) + C_2L_1'(\eta_1)L_1'(\eta_2)P_1(x) + C_3L_1'(\eta_1)L_1'(\eta_2)P_2(x) + C_4L_2'(\eta_1)L_2'(\eta_2)P_0(x)] \quad (5)$$

where $\eta_1 = 3.4\ r_1/a$, $\eta_2 = 3.4\ r_2/a$, x = cosine of the angle between the two radii drawn from the nucleus to the electrons; the functions $L_n'(\eta)$ and $P_m(x)$ have their usual meanings. The energy value was calculated without actually determining the C's. In order to find $\sum\bar{r}^2$, it becomes necessary, however, to evaluate them. Using the energy value and other data given by Kellner, we obtain the following:

$$C_1 = 14.336/a^3 \qquad C_2 = -0.832/a^3$$
$$C_3 = -0.082/a^3 \qquad C_4 = -0.113/a^3$$

Putting these values of C's into (5) and carrying out the integration, we find

$$\sum\bar{r}^2 = 1.917\ a^2 \quad \text{and} \quad X_M - 1.53 \times 10^{-6}.$$

Experimental determinations of X_M for helium have been made by Wills and Hector[4] and those for hydrogen by Wills and Hector,[4] Soné[5] and Lehrer.[6] We tabulate below their experimental values as well as the calculated ones:

	CALCULATED	OBSERVED	
He	-1.53×10^{-6}	-1.88×10^{-6}	W. and H.
H_2	-4.71×10^{-6}	-3.94×10^{-6}	W. and H.
		-3.99×10^{-6}	Soné
		-5.1×10^{-6}	Lehrer

The energy values found by applying Ritz method to the approximate ψ-functions used above agree with the spectroscopic determinations to 0.9% in the case of helium and to about 2% in the case of hydrogen molecule. If we assume that these percentage errors just represent the discrepancies between the approximate and the exact "Eigenwerte" of the wave equations, the errors in the $\sum\bar{r}^2$ obtained above may still be much higher because the error in the ψ-functions are probably higher. This

follows from the fact that when we look at the wave equation from the calculus of variations standpoint, we know that in the neighborhood of the correct "Eigenfunktion" ψ and the correct "Eigenwerte" E, $\delta E = 0$ for any $\delta\psi$. Incidentally, this also indicates why the Ritz method seems to be a particularly powerful one in calculating the approximate energy values of the wave equation.

[1] Van Vleck, J. H., *Proc. Nat. Acad. Sci.*, **12**, 662, 1926.

[2] An abstract of this paper has appeared in the Bulletins of *Amer. Phys. Soc.*, Chicago meeting, November, 1927.

[3] Kellner, G. W., *Zeits. Physik*, **44**, 91, 1927.

[4] Wills and Hector, *Physic. Rev.*, **23**, 209, 1926; Hector, *ibid.*, **24**, 418, 1926.

[5] Soné, T., *Phil. Mag.*, **39**, 305, 1920.

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ON THE QUANTUM THEORY OF THE POLARIZATION OF IMPACT RADIATION

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The general theory of the polarization of light excited by electronic impact has been given in a previous paper.[1] In particular, it is shown that the polarization computed on the quantum mechanics is independent of the axis chosen for describing the directional degeneracy of the atom. It is further shown that, if one neglects the spin of the impacting electron and if one assumes that the perturbing interaction energy is small, the polarization of the emitted light may be computed "as if" only those states were excited, for which the component of angular momentum in a certain direction is the same as in the normal state; and the direction is that of the vector difference of the velocities of the incident and scattered electron. If this result is applied to electrons with just the necessary energy to excite the atom, it yields the momentum rule formulated by Skinner; the excitation does not change the component of angular momentum in the direction of the electron beam. Experimentally[2] one finds that these rules fail in two ways: (1) They do not apply to intercombination lines; in particular, they fail conspicuously for radiation from the $^3S_1$ and $^3P_1$ states of mercury; and (2) the observed polarization tends, as the energy of the exciting electron is diminished, first, towards a maximum, which in the case of normal lines is proportional to and of the same sign as that given by the rule, and, as the energy is further decreased, tends to zero. In this paper we shall show that the first of these discrepancies

may be explained when one takes into account the spin of the impacting electron and the Heisenberg resonance principle;[3] and that the second follows when one discards the assumption that the perturbing energy is small. The theory may thus be made to give a complete account of the experimental results.

We shall first show that, as the velocity of the exciting electron approaches that of the resonance potential in question, the polarization of the excited line tends to zero. If $\psi_n(X)$ be the wave functions of the atom, we may write[4] the wave function for the system atom + electron:

$$\Psi(X; r, \vartheta, \varphi) = \sum_{n=0} \psi_n(X)\chi_n(r, \vartheta, \varphi) = \sum_{n=0} \sum_{p=0} \sum_{s=-p}^{s=+p} \psi_n(X) Y_{ps}(\vartheta, \varphi) y_{nps}(r) \quad (1)$$

where $$r^{-2}(r^2 y'_{nps})' + [\lambda_n^2 - p(p+1)r^{-2}]y_{nps} + \alpha_{nps}(r) = 0 \quad (2)$$

$$\alpha_{nps}(r) = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta d\vartheta Y^*_{ps}(\vartheta, \varphi) \sum_{n'=0} V_{nn'} \chi_{n'}. \quad (3)$$

Here $\frac{h^2\lambda_n^2}{8\pi^2 m}$ is the excess of the energy of the exciting electron over the resonance energy; r, ϑ, φ, are the polar coördinates of the electron with the polar axis chosen in the same arbitrary direction as for the atom; Y_{ps} are the normalized spherical harmonics; and V is the interaction energy of atom and electron. Whenever λ_n^2 is positive, y_{nps} will be the amplitude of the probability of the excitation of the state n, which is associated with that electronic wave which behaves asymptotically like

$$\frac{e^{-i\lambda_n r}}{r} Y_{ps}(\vartheta, \varphi). \quad (4)$$

Thus y_{n00} is the amplitude for the excitation which is independent of the initial direction of the electron; if $y_{nps} = 0$ for $p \neq 0$, the light emitted from the state n would be unpolarized. We have thus to show that

$$\lim_{\lambda_n \longrightarrow 0} y_{nps}/y_{n00} \longrightarrow 0 \qquad \text{For } p \neq 0,\ n \neq 0. \quad (5)$$

Now, uniformly in $\lambda_n \longrightarrow 0$, for $n \neq 0$,

$$\alpha_{nps} \leqslant A/r^{\alpha+\epsilon} \text{ for } r \geqslant r_0; \quad \alpha_{nps} \leqslant A/r \text{ for } r \leqslant r_0; \quad \epsilon > 0. \quad (6)$$

For the non-diagonal terms this is shown in l. c. 2. For the diagonal term $V_{nn}\chi_n$ it follows from the fact that the dipole moment of the atom has no diagonal elements; and this excludes only the case of atomic hydrogen, which has a first order Stark effect.

If we now use, as in l. c. 2, for the solution of (2),

$$y_{nps} = c_+ \xi^{-1/2} J_{+p+1/2}(\xi) + c_- \xi^{-1/2} J_{-p-1/2}(\xi) + I(\xi); \ \xi = \lambda_n r; \quad (7)$$

with $$I(\xi) = \int_{+\infty}^{\xi} dx\, G(x, \xi)\, \alpha_{nps}(x/\lambda_n); \tag{8}$$

$$G(x, \xi) = \frac{J_{+p+1/2}(x)\ J_{-p-1/2}(\xi) - J_{-p-1/2}(x)\ J_{+p+1/2}(\xi)}{J_{+p+1/2}(x)\ J'_{-p-1/2}(x) - J'_{+p+1/2}(x)\ J_{-p-1/2}(x)} \tag{9}$$

we find, for the asymptotic outgoing wave associated with that solution (F) which is everywhere finite, and which corresponds to no incident wave—since there are initially no atoms in the excited state n—

$$2c_{nps}/\xi \cdot e^{-i(\xi + p/2\pi)} \tag{10}$$

where $$c_{nps} = \lim_{\xi \longrightarrow 0} \int_{+\infty}^{\xi} dx\, F(x, \xi)\alpha_{nps}(x/\lambda_n) \tag{11}$$

$$F(x, \xi) = \frac{-G(x, \xi) \cdot \xi^{1/2}}{J_{-p-1/2}(\xi)}. \tag{12}$$

With $$\lim_{\xi \longrightarrow 0} F(x, \xi) \leqslant Ax^{\alpha+p} \quad \text{for } x \leqslant x_0; \tag{13a}$$

$$\lim_{\xi \longrightarrow 0} F(x, \xi) \leqslant Ax \quad \text{for } x \geqslant x_0; \tag{13b}$$

this gives

$$\lim_{\lambda_n \longrightarrow 0} c_{nps}/c_{n00} \sim \lambda^p \longrightarrow 0 \quad \text{for } p \neq 0. \tag{14}$$

This establishes our first point.

As was explained in l. c. 1,[5] the spin of the exciting electron is important only because of Heisenberg's resonance: there is a chance that the colliding electron will change places with one of the valence electrons of the atom. For normal lines this effect is not important, since in the excitation of these lines the magnitude of the total spin vector of the system will not change; the probability of interchange of electrons will thus be of the order of the interaction energy of the spins compared with that of the atomic spin and atomic orbit, and this will be negligible. It is thus permissible, for these cases, to couple the atomic spin and atomic orbital angular momentum before applying the perturbation; and this leads at once to Skinner's rules for the *total* angular momentum of the atom. For intercombination excitations the situation is different: the total spin vector can only be conserved if the spin of the impacting electron changes during the excitation. This form of excitation (A) will be more probable than that (B) in which the atomic coupling remains undisturbed, in the same way in which normal transitions are more probable the intercombination transitions in an atom. We may thus expect that when a state can be excited by (A), the momentum rules will fail, and that the correct

polarization must be computed by coupling spin and angular momentum *after* considering the perturbation; when (A) is impossible, the lines should be considerably weaker, and should be polarized in the same way, but not so completely, as predicted by the momentum rules.[6]

We shall now show how to compute the polarization of lines excited by scheme (A). If we treat the collision problem with neglect of all spins, we find[7] that the component of atomic orbital angular momentum parallel to the vector difference of velocities of incident and scattered electron remains unchanged by the excitation. We thus know k, s and the component in this direction, k_z, of k; what we want to know is the probability amplitude of $j, k, s, j_z = m$. For from these amplitudes, for given j and all m, we can calculate, by the methods of l. c. 1, the polarization of the spontaneous radiation from the state j, k, s. Thus we need only the transformation functions from the quantities $k, s, k_z, s_z = m - k_z$, to j, k, s, m. This problem can be solved by the canonical methods of the quantum mechanics. For if we can write j as a function of the quantities k, s, k_z, s_z, and other quantities whose laws of commutation with these are known, then the required amplitude will be given by

$$\{j(k', s', k_z', m' \ldots.) - j'\}(j', k', s', m'/k', s', k_z', m') = 0. \quad (15)$$

Now

$$j(j+1) = k(k+1) + s(s+1) - 2(k.s); \quad (k.s) = k_x s_x + k_y s_y + k_z s_z; \quad (16)$$

so that

$$j(j+1) = k(k+1) + s(s+1) - 2[k_x s_x + k_y s_y + k_z s_z]. \quad (17)$$

Instead of solving (15) directly as a matrix equation, we may, by repeated applications of the formula

$$(x'/z') = \int (x'/y')\, dy'(y'/z') \quad (18)$$

obtain $\gamma(j', k_z', m') = (j', k', s', m'/k', s', k_z', m') =$

$$\sum_{k_x', k_y', s_x', s_y', q_x', q_y'} (s_x'/s_z')(s_y'/s_z')(k_x'/k_z')(k_y'/k_z')\delta_{m'-k_z'}^{s_z'}\, \delta_{s_x' k_x'}^{q_x'}\, \delta_{s_y' k_y'}^{q_y'} \ldots.$$
$$\delta_{2q_x'+2q_y'+2k_z' s_z'}^{j'(j'+1)-s'(s'+1)-k'(k'+1)}. \quad (19)$$

The moduli of the functions

$$(s_x'/s_z') \ldots\ldots\ldots (k_y'/k_z')$$

are known, but their phases are indeterminate, and one must average over them in the final result. The phases of

$$(s_x'/s_z') \quad \text{and} \quad (k_x'/k_z')$$

are independent of each other, but those of

$$(s_x'/s_z'),\ (s_y'/s_z') \quad \text{and of} \quad (k_x'/k_z'),\ (k_y'/k_z')$$

are not. One has thus to average only over the independent phases of (s_x'/s_z') and (k_x'/k_z').

We may apply these results to the case of mercury, which has been studied in detail by Skinner.[2] Let us take first the simple case of the $^3S_1$ states. Here

$$k' = 0, \quad j' = s' = 1; \quad m' = \pm 1, 0;$$

and one readily finds

$$|\gamma(1, 0, 0)|^2 = |\gamma(1, 0, \pm 1)|^2 = 1. \tag{20}$$

The radiation from this state is thus unpolarized on scheme (A). If it were excited by (B), it would be completely polarized. Actually Skinner finds that the polarization is either absent or extremely weak.

Another case of interest is that of $^3P_1$, which gives the resonance line. Here $j' = k' = s' = 1; m' = 0, \pm 1;$

$$(0/0) = \int^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta d\vartheta \sin\vartheta \cos(\varphi + \beta) \cos\vartheta = 0 \tag{21}$$

so that $\gamma(1, 0, 0) = 0$. Further, (19) gives

$$|\gamma(1, 0, \pm 1)|^2 = {}^1/_2. \tag{22}$$

The polarization of the resonance line is thus of opposite sign and similar magnitude to that given by the momentum rules. That is what is found.[2] By the same methods one finds for the $^3P_2$ terms:

$$\left.\begin{aligned} &\gamma(2, 0, 0) = \gamma(2, 0, \pm 2) = 0, \\ &|\gamma(2, 0, \pm 1)|^2 = {}^1/_2. \end{aligned}\right\} \tag{23}$$

For the $^3D_1$ and $^3D_2$ states the excitation (A) is impossible. Lines from these states should, therefore, be weaker than the others, and their polarization should fall rather below that of the momentum rules. For $^3D_1$ this has been reported by Skinner.[2] For the $^3D_3$ the excitation (A) is possible; the states $m = 0, \pm 1$ are all excited, but the polarization is only slightly less than that of the momentum rules. This is also confirmed.

It may be mentioned that the transformation function $\gamma(j', k_z', m')$ enables one to answer the question: Given that an atom has a certain energy in a weak magnetic field; what is the probability that it will have a given energy (Paschen-Back level) in a strong field?

In conclusion we may summarize the theoretical predictions on the polarization of impact radiation: For very high electronic velocities the line has,[1] insofar as the excitation is direct (in principle this can be sepa-

rated from cascade effects), a polarization $-p$. As the velocity of the exciting electron is diminished, this polarization also diminishes, and changes sign at a voltage of the order of 200. The polarization then increases with diminishing velocity, and the curve, if extrapolated to the resonance velocity, gives a polarization $+p$. When the velocity of the electron approaches that of the resonance potential, the polarization grows more slowly, and has a maximum within a volt or so of the resonance potential; thereafter it approaches zero. The polarization p may, for normal lines, and for intercombination lines for which the excitation (A) is impossible, be computed by the momentum rule; for other intercombination lines it is more nearly given by the method (19) outlined above.

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[1] Oppenheimer, J. R., *Z. Physik*, **43**, 27, 1927. Referred to as l. c., 1.

[2] For the former discrepancy see Skinner, H. W. B., *Roy. Soc. Proc.*, **A112**, 642, 1926. I am much indebted to Dr. Skinner for informing me of his recent results by letter.

[3] L. c., 1, p. 46.

[4] See a paper on the Ramsauer effect, referred to as l. c., 2, to appear shortly in the *Z. Physik*.

[5] P. 45.

[6] The polarization should be less marked in part because of the relative importance of cascade excitation.

[7] L. c., 1, p. 38.

THE RESIDUAL SET OF A COMPLEX ON A MANIFOLD AND RELATED QUESTIONS (SECOND NOTE)

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1. In a recent Note under the same title[1] I communicated two very general formulas concerning certain topological simultaneous invariants of a C_k on an M_n, true subset of the manifold (formulas (3), (4) of the Note). The proofs were given explicitly and completely for a polyhedral C_k, i.e., a subcomplex of a subdivision of the C_n defining M_n. Regarding the general C_k it was stated on the strength of a certain theorem that the derivation of the formulas could proceed without change, and likewise for the most general closed subset G of M_n. On more searching examination it turned out to be decidedly otherwise, partly due to a necessary change in the theorem, and that the proofs for the polyhedral C_k would have to undergo profound change before they could carry over to the more general

types. Furthermore for G the type of cycle (Vietoris-cycle) considered, loc. cit., appears insufficient. We have, therefore, devised new proofs on somewhat different bases and which we can only give here in outline.[2] The proof of the second formula rests on a most interesting intersection theorem which we state below. The new definition of cycles and complexes on G that the case seems to require is likewise noteworthy and will be briefly described.

2. The theorem above referred to will be found loc. cit., p. 619. In its statement as well as in that of the corollary on the same page the "mod. N_δ" must be replaced by "mod. $N_{\delta'}$ where $\delta' \longrightarrow 0$ with δ."[3] This is due to the fact that the n-cells of M_n which arise in Alexander's construction may be of diameter $>\delta$, although their upper bound $\delta' \longrightarrow 0$ with δ.

3. The proof of (3) is essentially along the same lines as previously. We prove first that the numbers S_μ are finite, show that when δ is sufficiently small

$$S_\mu(M_n - C_k;\ M_n) = S_\mu(K - \Phi;\ K),$$

from which and (1) as applied to K and Φ follows (3).

For the second formula we first show that it is equivalent to

$$R_\mu(M_n - C_k) = \rho_{n-\mu},$$

the number of $n - \mu$ complexes of M_n with boundary (if any) on C_k independent mod. M_n upon the complexes on C_k itself. The proof of this last formula rests then upon the following intersection theorems in which Γ_μ is a cycle on $M_n - C_k$ and $C_{n-\mu}$ a complex with its boundary on C_k:

I. In order that $\Gamma_\mu \approx 0$ mod. $M_n - C_k$ it is necessary and sufficient that the Kronecker index $(\Gamma_\mu . C_{n-\mu}) = 0$ whatever $C_{n-\mu}$.

II. In order that $C_{n-\mu}$ depend mod. M_n upon a complex on C_k it is necessary and sufficient that the same index be zero whatever Γ_μ.

When C_k is a vacuous complex both theorems reduce to a proposition first stated by Poincaré and proved by Veblen and Hermann Weyl.[4] Similar propositions hold for an M_n with boundary and $C_k = C_{n-1}$ boundary of M_n and are implicitly proved in tr. 2.

The number $\rho_{n-\mu}$ is interesting for its own sake: If we disregard in all boundary relations complexes on C_k, we obtain a connectivity index which is precisely ρ.

4. Regarding the closed set G, the formulas for C_k can be derived in much the same way as for C_k, provided the notions of *complex on G* is suitably defined.

In the first place given any C_μ on M_n, we introduce the symbol $\frac{C_\mu}{t}$, t an integer $\neq 0$, and combine such symbols like ordinary fractions. Equal-

ity between them will be denoted by the symbol $\approx$ of homology with division allowed and we write down by definition $\frac{C_\mu}{t} \approx \frac{aC_\mu}{at}$, a an integer; $\frac{C_\mu}{t} - \frac{C'_\mu}{t'} \approx 0$ equivalent to $\frac{C_\mu}{t} \approx \frac{C'_\mu}{t'}$, and to $tC' - t'C_\mu \approx 0$, etc.

We consider next a sequence $\{N^i\}$ of closed neighborhoods of G, analogous to N of p. 619 of our first Note and such that every N^i carries the one that follows, while the maximum distance between points of N^i and $G \longrightarrow 0$ when $i \longrightarrow \infty$.

A μ-cycle on G is now defined as a sequence $\gamma_\mu = \left\{\frac{\Gamma^i_\mu}{t_i}\right\}$ with Γ^i on N^i, and with the property that to every p there corresponds a q such that $\frac{\Gamma^i_\mu}{t_i} - \frac{\Gamma^j_\mu}{t_j} \approx 0 \bmod. N^p$ when $i, j \geqq q$. We write $\gamma_\mu \approx 0 \bmod. M_n$ when under analogous conditions $\Gamma^i \approx 0 \bmod. M_n$. A similar type of sequence $c_\mu = \left\{\frac{C^i_\mu}{t_i}\right\}$, serves to define a complex with boundary on G, etc. Complexes and cycles give rise to homologies and related invariants entirely analogous to those for C_k. These invariants are independent of the choice of $\{N^i\}$, and indeed $R_\mu(G)$ is an invariant of G alone. We can define the Kronecker index $(\Gamma_\mu . c^i_{n-\mu})$, Γ_μ cycle of $M_n - G$, as equal to the number $\frac{1}{t_i}(\Gamma_\mu . C^i_{n-\mu})$ (notation as above), whose value is independent of i when it is above a certain limit. We thus have a suitable basis for derivation of our formulas for the closed set along practically the same lines as for C_k.

5. The necessity of the seemingly highly artificial type of cycles and complexes here introduced rests upon the following considerations. Let Γ_μ on M_n be dependent upon cycles whose points are all within a distance δ of G, so that $t\,\Gamma_\mu \backsim \bmod. M_n$ to a cycle of this type. Let t be the least number having that property. If t remains finite as $\delta \longrightarrow 0$, Vietoris cycles are sufficient for the purpose; if it $\longrightarrow \infty$ our more general type is needed.

[1] Same volume of these PROCEEDINGS, pp. 614–622. The notations of this first Note will be used here.

[2] They will be given in completeness in a paper to appear in the *Annals of Mathematics*.

[3] The necessity of making this change was pointed out to me by Alexandroff.

[4] See Veblen, *Trans. Amer. Math. Soc.*, **25**, 1923 (540–550); Weyl, *Revista Mat.*, **1923**; Lefschetz, tr. 2, p. 431.

POWER SERIES EXPANSIONS IN THE NEIGHBORHOOD OF A POINT ON A SURFACE

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1. *Introduction.*—Starting with a system of differential equations which define a surface except for a projective transformation, we present a general method of calculating the coefficients of a canonical expansion of one non-homogeneous projective coördinate of a point on the surface as a power series in the other two coördinates to any number of terms. And we compute for the first time the coefficients of the terms of the fifth order.

Then several canonical expansions obtained to terms of the fourth order by Darboux, Wilczynski, Green, Fubini and others are correlated and a general expansion is obtained to terms of the fifth order, which, by suitable specializations combined with certain minor modifications, yields all of these expansions. Finally some geometric applications of the analytic results are made.

2. *Computation.*—If the four homogeneous projective coördinates $x^{(1)}, \ldots, x^{(4)}$ of a point P on a non-degenerate non-ruled surface S in ordinary space are given as analytic functions of two independent variables u, v, and if the parametric net on S is the asymptotic net, then the functions x are solutions of a system of differential equations which, by suitable choice of proportionality factor, can be reduced to Fubini's canonical form

$$x_{uu} = px + \theta_u x_u + \beta x_v, \quad x_{vv} = qx + \gamma x_u + \theta_v x_v, \quad \theta = \log(\beta\gamma). \quad (1)$$

The coördinates of a point Q on S near P may be represented by Taylor's formula as power series in the increments Δu, Δv corresponding to displacement from P to Q. Then by means of equations (1) and the equations obtained therefrom by differentiation it is possible to express each of these series in the form $x_1 x + x_2 x_u + x_3 x_v + x_4 x_{uv}$, where $x_1, \ldots, x_4$ are power series in Δu, Δv which represent the local coördinates of Q referred to the covariant tetrahedron x, x_u, x_v, x_{uv}, with a suitably chosen unit point.

It is easy to compute power series in Δu, Δv for the non-homogeneous coördinates x, y, z of Q defined by placing $x = x_2/x_1$, $y = x_3/x_1$, $z = x_4/x_1$. From these series it is possible to compute an expansion for z as a power series in x, y. The best method of procedure seems to be to set z equal to a power series in x, y with undetermined coefficients and then to demand that the expansions for x, y, z in powers of Δu, Δv shall satisfy this equation identically in Δu, Δv as far as terms of any desired order, starting with the terms of order zero, and then determining the coefficients of the terms of orders one, two, three, etc., sequentially.

The result of carrying out the computations indicated, to terms of order five, is

$$z = xy - \frac{1}{3}(\beta x^3 + \gamma y^3) + \frac{1}{12}[\beta\psi_1 x^4 - 4\beta\psi_2 x^3 y - 6\theta_{uv} x^2 y^2 - 4\gamma\psi_1 xy^3 + \gamma\psi_2 y^4] + \frac{1}{60}[\beta c_0 x^5 + 5\beta c_1 x^4 y + 10\beta c_2 x^3 y^2 + 10\gamma c_3 x^2 y^3 + 5\gamma c_4 xy^4 + \gamma c_5 y^5] + \ldots \quad (2)$$

where

$$\psi_1 = \frac{\partial}{\partial u}\log \beta\gamma^2, \qquad \psi_2 = \frac{\partial}{\partial v}\log \beta^2\gamma$$

$$c_0 = \psi_1 \frac{\partial}{\partial u}\log\frac{\psi_1}{\beta\gamma} - \psi_1^2 + 8\beta\psi_2 - 4p, \quad c_1 = \psi_1\psi_2 - \psi_{2u} + 7\theta_{uv} + 4\beta\gamma, \quad (3)$$

$$c_2 = 3\gamma\psi_1 - \psi_2 \frac{\partial}{\partial v}\log \beta\psi_2 + 2q, \qquad c_3 = 3\beta\psi_2 - \psi_1 \frac{\partial}{\partial u}\log \gamma\psi_1 + 2p,$$

$$c_4 = \psi_1\psi_2 - \psi_{1v} + 7\theta_{uv} + 4\beta\gamma, \qquad c_4 = \psi_2 \frac{\partial}{\partial v}\log\frac{\psi_2}{\beta\gamma} - \psi_2^2 + 8\gamma\psi_1 - 4q.$$

If a covariant unit point is introduced by the transformation

$$x = l\bar{x}, \quad y = m\bar{y}, \quad z = n\bar{z},$$

where

$$l = \epsilon/\sqrt[3]{\beta^2\gamma}, \quad m = \epsilon^2/\sqrt[3]{\beta\gamma^2}, \quad n = 1/\beta\gamma, \quad \epsilon^3 = 1,$$

the expansion (2) becomes

$$z = xy - \frac{1}{3}(x^3 + y^3) + \frac{1}{12}[a_0 x^4 + 4a_1 x^3 y + 6a_2 x^2 y^2 + 4a_3 xy^3 + a_4 y^4] + \frac{1}{60}[b_0 x^5 + 5b_1 x^4 y + 10b_2 x^3 y^2 + 10b_3 x^2 y^3 + 5b_4 xy^4 + b_5 y^5] + \ldots, \quad (4)$$

where

$$a_0 = l\psi_1, \quad a_1 = -m\psi_2, \quad a_2 = -n\theta_{uv}, \quad a_3 = -l\psi_1, \quad a_4 = m\psi_2, \quad b_0 = mc_0/\beta, \quad b_1 = nc_1, \quad b_2 = lc_2/\gamma, \quad b_3 = mc_3/\beta, \quad b_4 = nc_4, \quad b_5 = lc_5/\gamma. \quad (5)$$

The new unit point, which in the original local system has coördinates $(1, l, m, n)$, may be characterized geometrically. This point lies on Fubini's canonical quadric,[1] $x_2x_3 - x_1x_4 = 0$, and lies on one of the three planes

$$\epsilon\beta^{1/3}x_2 - \gamma^{1/3}x_3 = 0,$$

determined by the projective normal and a tangent of Segre. And if the point be projected from the point (0, 0, 0, 1) onto the tangent plane

$x_4 = 0$, the projection, which will be denoted by U, has coördinates $(1, l, m, 0)$. The characterization will be completed by showing how to locate U by means of the cross ratio formed by U with three other points on the tangent of Segre. One of these is the point P with coördinates $(1, 0, 0, 0)$. A second is the point R, with coördinates

$$(0, \quad \gamma^{1/3}, \quad \epsilon\beta^{1/3}, \quad 0),$$

where the Segre tangent crosses the polar $x_1 = x_4 = 0$ of the projective normal $x_2 = x_3 = 0$ with respect to the canonical quadric. The third is the point T, with coördinates

$$[2\epsilon^2(\beta\gamma)^{2/3}, \quad \gamma^{1/3}, \quad \epsilon\beta^{1/3}, \quad 0],$$

where the Segre tangent crosses the cubic curve

$$x_4 = \beta x_2^3 + \gamma x_3^3 - x_1x_2x_3 = 0$$

corresponding to the projective normal by Segre's transformation. The point U is such that $(P, R, U, T) = 2$.

3. *Correlation of Different Expansions.*—The fact that two opposite edges of the tetrahedron of reference for the expansion (4) are the projective normal and its polar with respect to the canonical quadric of Fubini suggests that we may use instead any other two lines in the same polar relation, provided that one of them passes through P but does not lie in the tangent plane π, while the other lies in π but does not pass through P. The transformation which effects this change of tetrahedron is

$$x = (\bar{x} - A\bar{z})/D, \quad y = (\bar{y} - B\bar{z})/D, \quad z = \bar{z}/D, \quad D = 1 - B\bar{x} - A\bar{y} + h\bar{z},$$

the new unit point being related to the new edges of the new tetrahedron in the same way as the old unit point was related to the projective normal and its polar. Under this transformation the expansion (4) becomes

$$z = xy - \frac{1}{3}(x^3 + y^3) + \frac{1}{12}[(a_0 - 4B)x^4 + 4(a_1 + 2A)x^3y + 6(a_2 + 2AB - 2h)x^2y^2 + 4(a_3 + 2B)xy^3 + (a_4 - 4A)y^4] + \frac{1}{60}[(b_0 + 10a_0B - 20B^2 - 20A)x^5 + 5(b_1 + 12h - 2a_0A - 4AB + 4a_1B)x^4y + 10(b_2 - 2a_1A - 2A^2 - 2B)x^3y^2 + 10(b_3 - 2a_3B - 2B^2 - 2A)x^2y^3 + 5(b_4 + 12h - 2a_4B - 4AB + 4a_3A)xy^4 + (b_5 + 10a_4A - 20A^2 - 20B)y^5] + \ldots . \quad (6)$$

If the two reciprocal polar lines which we have just chosen for edges of the fundamental tetrahedron are canonical lines,[2] then

$$A = -mk\psi_2, \quad B = -lk\psi_1, \quad k = \text{const.},$$

and the expansion (6) becomes

$$z = xy - \frac{1}{3}(x^3 + y^3) + \frac{1}{12}[(1 + 4k)l\psi_1 x^4 - 4(1 + 2k)m\psi_2 x^3 y +$$
$$6(nk^2\psi_1\psi_2 - n\theta_{uv} - 2h)x^2y^2 - 4(1 + 2k)l\psi_1 xy^3 + (1 + 4k)m\psi_2 y^4] +$$
$$\frac{1}{60}\Big[\Big\{\psi_1 \frac{\partial}{\partial u}\log\frac{\psi_1}{\beta\gamma} - (20k^2 + 10k + 1)\psi_1^2 + 4(2 + 5k)\beta\psi_2 - 4p\Big\}l^2x^5 +$$
$$5\{(1 + 2k)(1 + 4k)\psi_1\psi_2 - \psi_{2u} + \theta_{uv} + 4\beta\gamma\}nx^4y + 10\Big\{(3 + 2k)\gamma\psi_1 -$$
$$\psi_2 \frac{\partial}{\partial v}\log\beta\psi_2 + 2q - 2k(1 + k)\psi_2^2\Big\}m^2x^3y^2 + 10\Big\{(3 + 2k)\beta\psi_2 -$$
$$\psi_1 \frac{\partial}{\partial u}\log\gamma\psi_1 + 2p - 2k(1 + k)\psi_1^2\Big\}l^2x^2y^3 + 5\{(1 + 2k)(1 + 4k)\psi_1\psi_2 -$$
$$\psi_{1v} + \theta_{uv} + 4\beta\gamma\}nxy^4 + \Big\{\psi_2 \frac{\partial}{\partial v}\log\frac{\psi_1}{\beta\gamma} - (20k^2 + 10k + 1)\psi_2^2 + 4(2 +$$
$$5k)\gamma\psi_1 - 4q\Big\}m^2y^5\Big] + \ldots. \quad (7)$$

If the vertex (0, 0, 0, 1) be chosen so as to lie on the canonical quadric of Wilczynski, whose equation is

$$x_2x_3 - x_1x_4 + \Big(AB - \frac{1}{2}\frac{\theta_{uv}}{\beta\gamma} - h\Big)x_4^2 = 0,$$

then the coefficient of x^2y^2 in expansion (6) vanishes. With this simplification and with the transformation of unit point

$$x = -\frac{1}{2}\bar{x}, \quad y = -\frac{1}{2}\bar{y}, \quad z = \frac{1}{4}\bar{z},$$

equation (7) is Wilczynski's canonical expansion[3] if $1 + 2k = 0$, and is Green's canonical expansion[4] if $1 + 4k = 0$. These geometers, however, carried their expansions only to terms of the fourth order and started with a different canonical form of the fundamental differential equations.

If we had started with the canonical form of the differential equations characterized by the conditions $p = q = 0$, then equation (7) would contain all of Fubini's canonical expansions.[5] He carries his expansions only to terms of the fourth order, however.

4. *Geometric Applications.*—We shall now recur to expansion (2) and make some geometric applications thereof, remarking that similar applications might be made of any of our other expansions.

First of all, we shall obtain conditions on the coefficients of equations (1) which are necessary and sufficient that S be a cubic. If we write the most general equation in x, y, z of a non-composite cubic surface and then

seek to determine the coefficients of this equation so that the cubic may have contact of the fifth order with S at P, we find that S must be itself a cubic, and that the required conditions, therefore, are

$$\frac{\partial^2}{\partial u \partial v} \log \frac{\beta}{\gamma} = 0,$$

$$p = \frac{1}{4} \psi_1 \frac{\partial}{\partial u} \log \frac{\psi_1}{\beta\gamma} + \frac{1}{16} \psi_1^2 - \frac{1}{12} \beta\psi_2, \tag{8}$$

$$q = \frac{1}{4} \psi_2 \frac{\partial}{\partial v} \log \frac{\psi_2}{\beta\gamma} + \frac{1}{16} \psi_2^2 - \frac{1}{12} \gamma\psi_1.$$

The algebraic surface of order n obtained by truncating the series (2) after terms of degree n will be called the canonical n-ic surface, and will be denoted by S_n. This is Fubini's canonical quadric if $n = 2$, and is his canonical cubic if $n = 3$. The general surface S_n may be characterized geometrically by two properties. First, S_n has a unode of order $n - 1$ at the point (0, 0, 0, 1), the unodal plane being the plane $x_1 = 0$; this means that the tangent plane and the polar surfaces of S_n at (0, 0, 0, 1) up to and including the surface of order $n - 2$ are indeterminate, while the polar surface of order $n - 1$ degenerates into the plane $x_1 = 0$ counted $n - 1$ times. Second, S_n has contact of order n with S at P.

If we denote by φ_k the sum of terms of degree k in (2), then the straight lines $x_4 = \varphi_n = 0$ are the tangents at P to the curve of intersection of S_{n-1} and S. The planes $\varphi_n = 0$ project these tangents from (0, 0, 0, 1) and, moreover, contain the entire intersection of S_{n-1} and S_n which, therefore, consists of n plane curves of order $n - 1$.

More generally, S_{k-1} and S_n intersect on a cone which has its vertex at (0, 0, 0, 1) and which intersects the tangent plane in a curve with a k-ple point at P, the k-ple point tangents being the lines $x_4 = \varphi_k = 0$. We shall denote this curve by $C_{k-1,n}$. In particular, $C_{1,n}$ is the intersection of S_n and the tangent plane, and $C_{1,2}$ consists of the asymptotic tangents. The curve $C_{n-2,n}$ seems to merit special consideration and can be characterized independently as follows. It is a curve of order n in the tangent plane with an $(n - 1)$-ple point at P, the $(n - 1)$-ple point tangents being the lines $x_4 = \varphi_{n-1} = 0$. It, moreover, passes through the intersections of the reciprocal of the projective normal, $x_4 = x_1 = 0$, with the planes $\varphi_n = 0$. The most general curve with these two properties has an equation of the form

$$x_4 = x_1\varphi_{n-1} + h\,\varphi_n = 0, \qquad h = \text{const.} \tag{9}$$

For $C_{n-2,n}$, h has such a value that the curve passes through the intersections of $C_{1,n-2}$ and $C_{1,n}$.

It is easy to show that each of the curves (9) crosses the asymptotic

tangents in two points, and that the line joining these points intersects $x_4 = x_1 = 0$ in a point that is independent of h, so that all of these lines form a pencil. In particular, if $n = 5$, the coördinates of the center of this pencil are $(0, -\psi_1 c_5, \psi_2 c_0, 0)$, and if $n = 4$ the center of the pencil is the canonical point $(0, -\psi_2, \psi_1, 0)$.

We have occasionally tacitly supposed $\psi_1\psi_2 \neq 0$. If $\psi_1 = \psi_2 = 0$, the surface S is such that all canonical lines through P coincide with the projective normal. Moreover, by a change of parameters we can make $\beta = \gamma = 1$. Then it is easy to see that S_3 has fourth order contact with S at P. Conversely, if S_3 has fourth order contact, and if S is not ruled, then $\psi_1 = \psi_2 = 0$.

[1] Fubini e Čech, *Geometria Proiettiva Differenziale*, Vol. I, p. 129.

[2] Fubini e Čech, loc. cit., p. 155.

[3] Wilczynski, "Second Memoir," *Trans. Amer. Math. Soc.*, **9** (1908), p. 98.

[4] Green, "Memoir on the General Theory of Surfaces, Etc.," *ibid.*, **20** (1919), p. 108.

[5] Fubini, Nuova trattazione elementare dei fondamenti della geometria proiettivo—differenziale di una superficia, *Rend. Reale Istituto Lombardo*, **59** (1926), p. 69.

THE ANALYSIS AND ANALYSIS SITUS OF REGULAR n-SPREADS IN (n + r) − SPACE

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The present note consists of a set of theorems and statements of methods, the elaboration of which will appear later in some other place.

1. *The Existence of Non-Degenerate Functions on n-Spreads without Boundary.*—We shall deal with n-spreads defined as follows. Let $(y) = (y_1, \ldots, y_{n+r})$ be rectangular coördinates in an euclidean $(n + r)$-space. $n > 1$, $r > 0$. Let $(u) = (u_1, \ldots, u_n)$ be rectangular coördinates in an auxillary n-space. By an *element* of an n-spread in our $(n + r)$-space will be meant a set of points homeomorphic with a set of points (u) within an $(n - 1)$ sphere in the space (u). By an *n-spread* $\sum_n$ *without boundary* will be meant a closed set of points (y), each one of which is on at least one of a finite set of elements and all points of which neighboring a given point of $\sum_n$ can be included in a single element of $\sum_n$.

An element will be said to be of *Class* C^n if the y's as functions of the u's are of class C^n, that is, possess continuous nth order partial derivatives. An element will be called *regular* if it is of at least class C', and at least one of the jacobians of n of the y's with respect to the u's is not zero, while $\sum_n$ will be called regular if capable of a representation by regular

elements. *By $\sum_n$ we shall always mean a regular, connected, n-spread of at least class C''.*

Let f be a single-valued, continuous function of the point on $\sum_n$. The function f will be said to be of *class C^r* on $\sum_n$, if as a function of any parameters (u) of a regular representation of $\sum_n$ of class C^r, f is of class C^r. *We shall deal only with functions of at least class C''.* A critical point (u_0) of f will be called *degenerate*, or *non-degenerate*, according as the hessian of f at (u_0) is or is not zero. A *function*, all of whose critical points are non-degenerate, will be termed *non-degenerate.*

THEOREM 1.—*On $\sum_n$ there exists infinitely many non-degenerate functions f of class C''.*

2. *Existence of Non-Degenerate Functions under Boundary Conditions.*—By a *regular region C_n* on $\sum_n$ will be meant a set of points on $\sum_n$ closed in our $(n + r)$-space and *bounded on $\sum_n$ by a set of regular $(n - 1)$ spreads B. We shall assume that $\sum_n$ and B are here of class C'''.*

THEOREM 2.—*Let the boundary B of the regular region C_n consist of two sets of $(n - 1)$ spreads, B' and B''. On C_n there exist infinitely many non-degenerate functions f of class C'', which take on over all of B' and B'' respectively, unique, non-critical, absolute maximum and minimum.*

3. *Relations between the Critical Points of a Function f.*—The earlier paper by the writer[1] dealt with an $(n - 1)$ region in n-space. The results of the earlier paper are extended to n-spreads such as $\sum_n$, and to n-regions on n-spreads. An entirely new development is the following study of critical points under general boundary conditions.

We deal here with the n-region C_n and boundary B described above. Let f_n be the derivative of f at B with respect to the arc length n along a regular curve exteriorly normal to B. On B let f equal a function φ. Let the existence and types of critical points of φ be determined in terms of φ as a function of the parameters of B.

THEOREM 3.—*Let there be given on C_n a non-degenerate function f of class C''' defining a non-degenerate function φ on B, the boundary of C_n. Suppose f is without critical points on B. Let M_i be the sum of the numbers of critical points of type i of f, and of φ at the points at which $f_n < 0$, and let R_i be the ith connectivity[2] of C_n. Between the numbers M_i and R_i the $(n + 1)$ relations of the earlier paper still hold true.*[3]

That the author's relations are the only relations involving the M_i's alone is proved by the following theorem.

THEOREM 4.—*If there be given any set of integers M_i satisfying the author's $(n + 1)$ relations[3] for the case of an n-region C_n homeomorphic to an n-simplex, then there exists a function, non-degenerate and of class C'' on C_n, constant on C_n's boundary, and possessing critical points of type i, in number equal to M_i.*

THEOREM 5.—*Let there be given on the regular region C_n in $(n + r)$-*

space a non-degenerate function f of class C'', with $f_n < 0$ on B, the boundary of C_n. Then between the numbers M_i of f, and the connectivities B_i of B, there exist the following n − 1 relations.

$$1 + M_n + (B_0 - 1) \leqq [M_0 + M_{n-1}]$$
$$1 + M_n + (B_0 - 1) - (B_1 - 1) \geqq [M_0 + M_{n-1}] - [M_1 + M_{n-2}]$$
$$1 + M_n + (B_0 - 1) - (B_1 - 1) + (B_2 - 1) \leqq [M_0 + M_{n-1}] - [M_1 + M_{n-2}] + [M_2 + M_{n-3}],$$

. . . . . .

where the signs < and > alternate successively. For n odd we have also

$$E = 2[M_0 - M_1 + \ldots - M_n] \qquad (1)$$

where E is the Euler-Poincaré characteristic of B.

4. *Relations between the Connectivities of a Regular Region C_n and Its Boundary B.*—Let f be a non-degenerate function f of class C'' on $\sum_n$. From this point on we shall need to use only functions f which take on no critical value at more than one critical point. A function f satisfying all these restrictions will be termed *admissible.*

THEOREM 6.—*Suppose there is on $f = c_1$, a single non-degenerate critical point of type k. Then corresponding to a sufficiently small increase of c through c_1, the connectivities of $f = c$ are unaltered except at most for the following changes:*

$k = 0$		$\Delta B_0 = \Delta B_{n-1} = 1$
$k = n$		$\Delta B_{n-1} = \Delta B_0 = -1$
$k = 1,2, \ldots, n-1$ $2k \neq n-1, n, n+1$	either or	$\Delta B_k = \Delta B_{n-k-1} = 1$ $\Delta B_{k-1} = \Delta B_{n-k} = -1$
$2k = n-1$	either or	$\Delta B_k = \Delta B_k = 2$ $\Delta B_{k-1} = \Delta B_{k-1} = -1$
$2k = n$	either or or	$\Delta B_k = \Delta B_{k-1} = 1$ $\Delta B_{k-1} = \Delta B_k = -1$ $\Delta B_k = \Delta B_{k-1} = 0$
$2k = n+1$	either or	$\Delta B_k = \Delta B_{k-2} = 1$ $\Delta B_{k-1} = \Delta B_{k-1} = -2$

These relations are proved by a resort to matrices.

A critical point of an admissible function will be said to be of *increasing* or *decreasing* type, respectively, relative to $f \leqq c$, according as a small increase of c through the given critical value causes an increase or decrease of the connectivities of $f \leqq c$ (see theorem 3, p. 385, Morse). Similarly, a critical point of f will be said to be of *increasing, decreasing* or *neutral* type relative to $f = c$, according as a small increase of c through the given

critical value causes an increase, decrease, or no change in the connectivities B_1, in accordance with the preceding table.

THEOREM 7.—*If a critical point is of decreasing type relative to $f \leqq c$ it will be of decreasing type relative to $f = c$.*

Examples will show that if a critical point is of increasing type relative to $f \leqq c$ it may be of increasing, decreasing, or even neutral type, relative to $f = c$. To pass from one critical point to a set of such points let a and b be two non-critical values of f. In the domain,

$$a \leqq f \leqq b, \tag{1}$$

let q_i and r_i be the number of critical points of type i, respectively, of decreasing and neutral type, relative to $f = c$; and Q_i the number of critical points of type i of decreasing type relative to $f \leqq c$.

THEOREM 8.—*Between the changes ΔB_1 in the connectivities of $f = c$, and the changes in the connectivities of $f \leqq c$, as c increases from a to b there exist the following n relations.*

$$\Delta\,[(R_i + R_{n-i-1} - B_i) - (R_{i-1} + R_{n-i} - B_{i-1}) + \ldots + (-1)^i (R_0 + R_{n-1} - B_0)] = q_{i+1} - Q_{i+1} + q_{n-i-1} - Q_{n-i-1} + r_{i+1} = 0.$$
$$i = 0, 1, \ldots, n-1.$$

THEOREM 9.—*Between the connectivities R_i and B_i, respectively, of any regular n-region and boundary of class C''' in euclidean $(n + r)$-space, there exist the following n relations:*

$$0 \leqq [R_i + R_{n-i-1} - B_i - 1] - + \ldots + (-1)^i [R_0 + R_{n-1} - B_0 - 1]$$
$$i = 0, 1, \ldots, n-2$$

$$0 = [R_0 + R_{n-1} - B_0 - 1] - + \ldots + (-1)^{n-2} [R_{n-2} + R_1 - B_{n-2} - 1] + (-1)^{n-1} [R_{n-1} + R_0 - B_{n-1}].$$

COROLLARY. *Under the same hypotheses,*

$$\begin{aligned} R_i + R_{n-i-1} &\geqq B_i + 1 \qquad i = 0, 1, \ldots, n-2 \\ R_{n-1} + R_0 &\geqq B_{n-1}. \end{aligned} \tag{2}$$

THEOREM 10.—*A necessary and sufficient condition that the regular n-spread $\sum_n$ in $(n + r)$-space have the connectivities of an n-sphere is that each critical point of any admissible function f be of increasing or decreasing type relative to $f \leqq c$, when and only when each such critical point is of increasing or decreasing type, respectively, relative to $f = c$.*

5. *Complimentary Regions.*

THEOREM 11.—*If C_n and C'_n are regular regions on $\sum_n$, such that $C_n + C'_n = \sum_n$, and if C_n, C'_n, and $\sum_n$ have the connectivities R_i, R'_i, and S_i, respectively, then*

$$(R'_{n-i-1} - R_i) - (R'_{n-i} - R_{i-1}) + \ldots + (-1)^i (R'_{n-1} - R_0)$$
$$\leqq (S_{n-i-1} - 1) - (S_{n-i} - 1) + \ldots + (-1)^i (S_{n-1} - 1)$$
$$i = 0, 1, \ldots, n-1.$$

If $\sum_n$ has the connectivities of an n-sphere we can prove that the inequalities of the preceding theorems all become equalities. We have the following corollary.

COROLLARY.—*If $C_n + C_n' = \sum_n$ and $\sum_n$ has the connectivities of an n-sphere, then*

$$R_i = R'_{n-i-1}, \qquad i = 0, 1, \ldots, n-1.$$

Alexander[4] has proved this corollary for the case where $\sum_n$ is actually an n-sphere, as distinguished from our hypothesis that $\sum_n$ be any regular n-spread in $(n + r)$-space with the connectivities of an n-sphere.

[1] Marston Morse, *Trans. Amer. Math. Soc.*, **27**, 1925, 345.

[2] Oswald Veblen, *The Cambridge Colloquium. Analysis Situs*, Chap. III.

[3] Morse, l. c., theorem 8, p. 392.

[4] F. W. Alexander, *Trans. Amer. Math. Soc.*, **23**, 348 (1922).

ON CERTAIN PROPERTIES OF FREQUENCY DISTRIBUTIONS OF THE POWERS AND ROOTS OF THE VARIATES OF A GIVEN DISTRIBUTION

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Given a continuous frequency function $y = f(x)$, $f(x) \geqq 0$ with a range from $x = a$ to $x = c$, where $0 < a < c$. Assume further that $f(a) = f(c) = 0$, and that the given distribution is unimodal with the mode at $x = b$, $a < b < c$.

Let us make a transformation by replacing each variate x by u where $u = x^n$ and n is a real number on which appropriate limitations will be placed as we proceed. We shall consider only the positive value of u when n is such that u may have more than one value corresponding to an assigned value of x.

In a previous paper,[1] the writer dealt with the frequency distributions that arise from applying the transformation $u = x^n$ to a set normally distributed variates with the origin so selected that the modal value, say $x = b$, was positive, and in cases of particular interest equal to at least three or four times the standard deviation.

By the transformation in question, a variate at the mode $x = b$ of $y = f(x)$ is carried into $u = b^n$. It is easily shown in the case of the normal distribution that the modal value of the distribution of transformed variates is greater than b^n when n is between 0 and 1, less than b^n when

n is outside this interval and, of course, equal to b^n in the trivial case in which $n = 1$. In the present note, it is shown first (theorem I) that this property of the distribution obtained by replacing each variate of the normal distribution by its nth power is a property of distributions of the nth power of variates of a large class of distributions. The remainder of the paper is concerned with the comparative values of the mean, median, and mode, and with the direction of the skewness of the distribution of $u = x^n$ when the frequency curve $y = f(x)$ is symmetrical about a line $x = b$. Certain limitations on the distribution $y = f(x)$ are stated at the beginning. Other limitations will be imposed as we proceed. It is, however, not unlikely that some of these limitations are unnecessary.

Given $u = x^n$

$$du = nx^{n-1}\,dx, \text{ except at } x = 0 \text{ when } n < 1.$$

$$dx = \frac{du}{nu^{(n-1)/n}}, \text{ except at } u = 0 \text{ when } n > 1.$$

Then the frequency function of u variates obtained from that of the x-variates is given by

$$v = \frac{1}{n}\,u^{(1-n)/n}\,f(u^{1/n}) \tag{1}$$

when n is positive.

We shall now prove the following:

THEOREM 1.—*Given a unimodal continuous frequency function $y = f(x)$ of positive variates with a range a to c, with a mode at $x = b (a < b < c)$, with $f(a) = f(c) = 0$, and with the derivative $f'(x)$ continuous, then the frequency distribution $v = \phi(u)$ of positive variates $u = x^n$ has a mode at a point $u > b^n$ when $0 < n < 1$, and at a point $u < b^n$ when n is outside the interval 0 to 1.*

To prove this theorem, we have from (1)

$$\frac{dv}{du} = \frac{u^{(1-2n)/n}}{n^2}\,[(1-n)f(u^{1/n}) + u^{1/n}f'(u^{1/n})] = 0 \text{ at a mode.} \tag{2}$$

We shall find it convenient to consider the frequency curves given by (1) under three cases according as $n > 1$, $0 < n < 1$, or $n < 0$.

CASE I.—$n > 1$. In this case, a simple examination of (2) shows dv/du is negative for every value of u such that $c^n > u \geqq b^n$. Hence, if a mode exists, it is at a point where $u < b^n$. Under the limitations placed on $f(x)$ at the start, we may prove as follows that a mode of $v = \phi(u)$ occurs in the interval a^n to b^n. We have seen that dv/du is negative when $u = b^n$. When $u = a^n$, $\frac{dv}{du} = \frac{a^{2-2n}}{n^2}\,f'(a)$, which is either positive or

zero. If it is positive, there is clearly a maximum at the point where the sign of the continuous derivative changes from positive to negative between $x = a^n$ and $x = b^n$. If the derivative dv/du is zero at $u = a^n$ it follows that there is at least one maximum of $v = \phi(u)$ between $u = a^n$ and $u = b^n$ since $v = 0$ at $u = a^n$ and then increases before decreasing at $u = b^n$.

CASE II.—$0 < n < 1$. In this case, a simple examination of (2) shows that dv/du is positive for every value of u such that $a^n < u \leqq b^n$. It follows from considerations similar to those presented under case I that a mode occurs at a point where u is between b^n and c^n. It is easy to verify this fact for certain important special types of frequency functions by actually finding the mode of $v = \phi(u)$. For example, consider the Pearson frequency curve $y = y_0 e^{-\gamma(x-2a)}(x - a)^{\gamma a}$ of type III with γ positive and with the mode at $x = b = 2a$.

Then (2) becomes

$$\frac{dv}{du} = \frac{v}{nu(u^{1/n} - a)} [(1 - n)(u^{1/n} - a) + 2a\gamma u^{1/n} - \gamma u^{2/n}]. \quad (3)$$

From (3), we have

$$u^{1/n} = \frac{2a\gamma + 1 - n \pm \sqrt{(1 - n)^2 + 4a^2\gamma^2}}{2\gamma} > 2a$$

when $n < 1$ if the positive sign is taken before the radical.

It is easily shown that the value given by taking the negative sign is outside the range of distribution. The value with the positive sign gives the mode at a point

$$u = \frac{(2a\gamma + 1 - n \pm \sqrt{(1 - n)^2 + 4a^2\gamma^2})^n}{(2\gamma)^n} > (2a)^n \text{ when } n < 1$$

CASE III.—$n < 0$. Take $n = -m$ where m is positive. Then the transformed positive frequency function is

$$v = \frac{1}{m} u^{(-1-m)/m} f(u^{-1/m})$$

$$\frac{dv}{du} = -\frac{u^{(-1-2m)/m}}{m^2} [(1 + m) f(u^{-1/m}) + u^{-1/m} f'(u^{-1/m})] = 0.$$

When $u = 1/b^m$, dv/du is clearly negative. When $u = 1/c^m$, dv/du is either zero or positive. From these facts and from conditions of continuity, it follows that a mode exists at a point where u is between $1/c^m$ and $1/b^m$.

The theorem thus established may be stated in a somewhat less formal manner by saying that the theoretical value occurring most frequently among the nth powers of the variates of the distribution $y = f(x)$ is greater or less than the nth power, b^n, of the most frequently occurring variate in the distribution $y = f(x)$ according as n is between 0 and 1 or outside of this interval.

Since a transformation of $u = x^n$ carries the median of a given distribution $y = f(x)$ into the median value of u in the distribution $v = \phi(u)$, we have the following:

THEOREM 2.—*In case the frequency curve $y = f(x)$ is symmetrical about the modal line $x = b$, the frequency distribution $v = \phi(u)$ of $u = x^n (n \neq 0$ nor 1) has a modal value greater or less than its median according as the value of n lies between 0 and 1 or outside of these bounds.*

Since the mean of u or the centroid $\bar{u}$ of area under the transformed curve $v = \phi(u)$ is given by

$$\bar{u} = \frac{1}{n}\int_{a^n}^{c^n} u^{1/n} f(u^{1/n})du = \int_a^c x^n f(x)dx,$$

we have $u = \mu_n'$, where μ_n' is the nth moment about the origin of the original frequency function $y = f(x)$. Denoting the mean value of x by $\bar{x}$, it is known[2] for every set of positive values that $\mu_n' < x^n$, when n lies between 0 and 1, and $\mu_n' > \bar{x}^n$ when n lies outside of the interval 0 to 1.

Since $\bar{x} = b$ when the function $y = f(x)$ is symmetrical about the line $x = b$, we have the following:

THEOREM 3.—*In case the frequency curve $y = f(x)$ is symmetrical about the modal line $x = b$, the mean value of u (centroid $\bar{u}$) in the frequency distribution $v = \phi(u)$ of $u = x^n$ ($n \neq 0$ nor 1) is less or greater than its median value according as the value of n lies between 0 and 1 or outside of these bounds.*

From theorems 2 and 3, we readily obtain the following corollary. *In case the frequency curve $y = f(x)$ is symmetrical about the modal line $x = b$, the frequency distribution $v = \phi(u)$ of $u = x^n$ ($n \neq 0$ nor 1) has negative or positive skewness in the Pearson sense according as the value of n lies between 0 and 1 or outside of these bounds.*

[1] *Ann. Math.*, **23**, No. 4 (1922), pp. 292–300.

[2] J. L. W. V. Jensen, *Acta Mathematica*, **30** (1906), pp. 180–187.

THE PENETRATION OF METHYLENE BLUE INTO LIVING CELLS

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The purpose of this paper is to show:

1. That the amount of methylene blue found in the sap of *Valonia* at equilibrium is independent of the pH value of the external solution.

2. That the rate of penetration is influenced by temperatures and pH value of the external solution.

3. That methylene blue penetrates into the sap of *Valonia* as such and not in the form of a lower homolog, trimethyl thionine or methylene azure.

In preliminary experiments[1] on the penetration of methylene blue into living cells of *Valonia*, it was noted that this dye penetrated into the sap at any pH of the external solutions from 5.4 to 9.0. The writer has had opportunity to extend these observations. The same methods were used and the dye was taken from the same lot[2] as in the former experiments. However, the temperature of the former experiments was 25°C., while in these experiments both 25°C. and 22.5°C. were used. The lower temperature made it possible to decrease the rate of penetration so as to make more detailed observations possible. The plants were placed in buffered solutions of methylene blue dissolved in sea water whose pH value ranged from 5.8 to 9.0. Clark's phosphate and borate buffers were used, keeping the concentration of buffer constant. At intervals, plants were taken out and the sap expressed. The concentration of the dye was measured by comparison with standardized solutions of dye contained in Pyrex tubes of the same size as were used for the expressed sap, and its identity was determined by means of spectrophotometric analyses.

In order to see whether the plants were injured, some of them were taken from the dye solution at various intervals, transferred to sea water, and kept for observation as to time of survival. That these plants were not injured has been many times proved by noting that they lived as long as the controls which had not been in the dye solution previously. They were always kept for observation at least two weeks. Observing them for one or two days only, after experimentation, is not long enough to note whether they are injured, as many cells appear normal for the first day or so after experimentation before they cytolyze.

The results were as follows:

At 25°C. penetration during the first hour was so fast that there were no distinguishable differences in the amount of dye in the sap of plants

exposed to dye solutions of different pH values from 5.8 to 9.0. This confirms the writer's previous observations,[1] and indicates that equilibrium of the dye in the sap is independent of the pH value of the external solution. At lower temperatures, however, as at 22.5°C., the penetration was sufficiently slowed up so that time curves could be followed. At this temperature it could be noted that the penetration increased as the pH value of the external solution increased. After four or five hours, however, the amount of dye in the sap, when the external pH values were low, equalled the amount which had accumulated in the sap in less time when the external pH was 9.0. Several successive readings after this seemed to indicate that an equilibrium had been reached before injury set in.

At temperatures lower than 21°C. the time necessary for penetration is so considerable that injury due to pH may result before the dye penetrates.

The spectrophotometric findings of the writer lead to important conclusions. It was found that when the sap of plants which had been placed in solutions of methylene blue at any pH from 5.8 to 9.0, from one to three hours, was expressed and *immediately* analyzed by the spectrophotometer, it gave a wave-length of maximum absorption characteristic for methylene blue with a primary maximum at 663 mμ and a secondary maximum at 610 mμ. When the same sap was examined *after having stood for some hours* it gave the absorption spectrum of trimethyl thionine, with a maximum at 650 mμ and a secondary maximum at 590 mμ. This shows definitely that methylene blue penetrates *Valonia* as such and that it gradually oxidizes in the sap when exposed to air, forming lower homologs, notably trimethyl thionine. It is necessary, therefore, to analyze the sap containing dye immediately as it comes out of the cell before it becomes oxidized. Irwin[3] states that it is not methylene blue which penetrates living cells of *Valonia* but trimethyl thionine formed in solutions of methylene blue. She bases this conclusion on spectrophotometric analysis of sap which gave the wave-lengths of maximum absorption for this homolog at 650 mμ, with a secondary maximum at 590 mμ. Her results indicate, however, that her readings were not made immediately after the sap was expressed, and that it seems probable that some time may have elapsed between the expression of the sap (which was evidently done in New York) and the spectrophotometric readings which were made in Washington, D. C., so that the time interval enabled the dye to become oxidized in transit.

The dye must be sufficiently dilute in the expressed sap in order to be completely oxidized to its lower homologs. More concentrated solutions of dye fail to show the change. This is very probably due to the inability of the sap to transform more than a given amount of dye. In some cases when the dye was extremely dilute, the absorption spectrum indicated the formation of even lower homologs.

A similar case showing oxidation of a dye in freshly expressed sap has

been discussed by the writer in connection with another oxidation-reduction dye, namely, 2-6-dibromo phenol indophenol.[1] This penetrates the sap in a reduced or colorless form and is oxidized to a blue form when the sap is exposed to the air.

This is a preliminary report. Details will be published subsequently. Grateful acknowledgment is hereby made to the BACHE FUND of the NATIONAL ACADEMY OF SCIENCES for a grant which enabled the writer to carry on this work, and to the members of the Physics Department of Rutgers University who kindly assisted in the use of the spectrophotometer.

[1] Brooks, M. M., *Amer. Jour. Physiol.*, **76**, 360, 1926.

[2] This dye is taken from a very pure sample made by Dr. W. M. Clark, to whom the writer is greatly indebted for the presentation of the dye.

[3] Irwin, M., *Jour. Gen. Physiol.*, **10**, 927, 1927.

STUDIES ON THE EFFECT OF POTASSIUM ALUM-HYDROCHLORIC ACID SOLUTIONS ON THE GROWTH AND FATE OF NEOPLASTIC TISSUE

II. Result Obtained on a Rapidly Growing Transplantable Sarcoma of the Mouse

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Since we were able to bring about a slowing up of the growth rate of a transplanted adenocarcinoma by administering a mixture of potassium alum-hydrochloric acid solution in the drinking water of the inoculated animals,[2] it would be of interest to determine whether a similar phenomenon could be obtained on other histologically or physiologically distinct tumors. For this reason we have employed a rapidly growing sarcoma of the mouse.

The tumor used in the experiments recorded in this paper is non-specific; that is, it grows equally well in all mice inoculated irrespective of the genetic stock to which the individuals belong. It grows very rapidly, reaching a mass equal in weight to the weight of the mouse harboring it, in from twenty-two to thirty days after inoculation. Death occurs when this point is reached. Recently, however, due to complicating factors in my laboratory, such as the presence of mites and pneumonia, death usually occurs before the tumor reaches a weight equivalent to the weight of the experimental animal.

The transplantable tissue used in the present experiment, therefore,

differs in several fundamental respects from the first inoculable tumor tissue used in similar experiments. (1) The present tumor is non-specific whereas the first tumor possessed a high degree of tissue specificity. By a long series of experiments extending over a period of nine years, we have been able to demonstrate, quite conclusively, that tissue specificity is a genetic problem. Since tissue specificity apparently has no bearing upon the present experiments, this phenomenon will not be considered here. (2) The present sarcoma is a rapid grower, growing at least from five to eight times faster than the adenocarcinoma previously used. (3) It is, of course, of fundamentally different histological structure.

Method.—As in the previous series of experiments, administration of the alum-hydrochloric acid solution has been by means of placing it in the drinking water of the mouse. Great difficulty was again experienced by not getting the animals to take the solution readily. We, therefore, resorted to the second method of administration outlined in the previous paper—that of starting with a large initial dosage and then gradually reducing it from day to day as the inoculated animals lost weight.

Results.—The results obtained with the transplantable sarcoma are given in table 1. As in the previous case of the adenocarcinoma, considerable variation in the dosage and percentage of the two chemicals was employed. For clearness in presentation, however, these various data are grouped together wherever possible. It will be noted (1) that in very nearly all cases the experimental animals lived, on an average, a few days longer than the controls; (2) that the experimental animals weighed less than the controls at the time death occurred, although in the two classes the animals weighed the same when the experiments were started, and (3) that, at the time of death of the inoculated animals, the weights of the tumors in experimental animals were smaller than those in the control mice.

In order to do some comparative estimates on the control and experimental animals, a few quotients were computed. The first value (T/D) was obtained by dividing the weight of the tumor at the end of the experiment with the time expressed in days that the individual mice lived after inoculation. This value represents the average growth of the inoculated tissue per unit of time. It must be borne in mind, however, that the growth of the transplant is logarithmic—an increasingly greater rate of growth occurring from day to day so that the value given does not mean or represent the rate of growth at any intermediate point.

The second value ($T/D//M$) was obtained by dividing the averaged transplanted tumor growth rate by the weight of the mouse at the end of the experiment. It, therefore, represents the average increment in weight of the tumor, per unit of time, per weight of the mouse. In all cases weights are expressive in grams.

General Discussion.—The corresponding values of the experimental animals and the controls are quite parallel. In most cases, however, the values for the experimental animals are somewhat lower than the corresponding values obtained on the controls, but these differences are probably not significant. Due to the difficulty in administration of the material, it is, however, not very important to increase the numbers for the sake of determining greater significance. This difference between experimentals and the controls may be due to the fact that the experimental animals consume less water during the experiments, due to the extreme bitterness of the alum solution. This lessened water intake undoubtedly influenced the weight of the mice during experimental conditions. It is not definitely determined, however, that a reduced body weight of the host harboring the transplant would materially affect the weight of the inoculated tumor. In some cases, such as the presence of infectious disease, etc., both the weight of the host and the weight of the transplant in a given time period are reduced below normal. On the other hand, I determined some years ago, on this same sarcoma, that younger animals (having, of course, a smaller body weight) would grow the transplant more rapidly than adult mice. This result, however, may have been influenced more by the age factors involved than the body-weight factors. The present experiments point out, at least, that more work ought to be done on these factors influencing the growth rate of inoculated tumors.

It is probable, therefore, that the alum solution has not affected the growth rate of the transplanted sarcoma. From previous experimentations it has been determined that the same material administered to mice harboring slow-growing transplantable adenocarcinoma will materially cause a slowing up of the growth of the transplant with a materially increased length of life of the inoculated individual.

It is very difficult to determine whether this difference between transplantable sarcoma and transplantable adenocarcinoma be produced by the relative difference in the normal growth rates of the two transplants, or due to the fundamental difference in the histological structure and physiological behavior of the two. Both of these biological forces may undoubtedly be functioning. This problem is being investigated further with a series of experiments on other types of transplantable tumors.

Conclusion.—A mixture of potassium alum-hydrochloric acid administered in the drinking water of the mouse does not materially influence the growth rate of a rapidly growing sarcoma.

Table showing the results obtained by the administration of a constantly reduced dosage of a mixture of potassium alum-hydrochloric acid in the drinking water upon the growth of a transplanted sarcoma. Survival days indicate the number of days the inoculated animals lived after in-

TABLE 1

NO. EXP.	DATE 1926	NO. MICE	SURVIVAL DAYS	WEIGHT MOUSE	WEIGHT TUMOR	*T/D*	*T/D//M*	NO. MICE	DAYS TREATMENT STARTED	MAXIMUM INITIAL DOSAGE DROPS	SURVIVAL DAYS	WEIGHT MOUSE	WEIGHT TUMOR	*T/D*	*T/D//M*	*E-C T/D*	*E-C T/D//M*
1	8/7	5	29.2	18.1	16.1	0.551	0.030	6	6	2	27.7	17.1	13.5	0.492	0.029	−0.059	−0.001
2	8/18	5	23.2	17.4	10.7	0.461	0.027	6	−14	3	24.0	14.0	9.7	0.404	0.029	−0.057	+0.002
3	9/11	5	28.2	16.7	16.1	0.571	0.034	4	1	4	31.8	16.2	13.3	0.418	0.026	−0.153	−0.008
4	9/20	5	21.2	16.4	10.8	0.509	0.031	6	4	5	24.0	15.1	8.9	0.371	0.025	−0.138	−0.006
5	9/30	5	20.8	17.7	11.7	0.563	0.032	5	4	6	25.4	17.5	10.3	0.406	0.023	−0.157	−0.009
6	10/16	6	30.8	13.6	9.9	0.321	0.024	5	1	6	23.0	13.8	7.6	0.330	0.024	+0.009	0.000
7	10/16	5	27.8	16.7	11.0	0.396	0.024	6	1	7	28.5	14.4	11.6	0.407	0.028	+0.011	+0.004
8	10/16	4	23.7	18.1	12.5	0.527	0.029	2	1	8	27.0	14.6	9.1	0.337	0.023	−0.190	−0.006
9	10/26	6	26.5	15.4	15.3	0.577	0.038	5	1	9	28.8	14.1	11.7	0.406	0.029	−0.171	−0.009
10	11/6	5	24.4	13.0	13.1	0.537	0.041	4	1	10	26.8	16.8	10.9	0.409	0.024	−0.128	−0.017
11	11/6	6	25.7	18.1	12.1	0.471	0.026	6	1	11	27.0	15.5	10.8	0.400	0.026	−0.071	0.000
12	11/6	4	21.7	15.2	7.4	0.341	0.022	4	1	12	26.8	13.6	6.4	0.239	0.018	−0.102	−0.004
				AVERAGES								AVERAGES					
1–3		15	26.9	17.4	14.3	0.532	0.031	16		3	27.8	15.8	12.2	0.438	0.028	−0.094	−0.003
4–6		16	24.3	15.9	10.8	0.444	0.028	17		5.7	24.1	15.5	8.9	0.370	0.024	−0.074	−0.004
7–9		15	26.0	16.7	12.9	0.496	0.030	13		8	28.1	14.4	10.8	0.385	0.027	−0.111	−0.003
10–12		15	23.9	15.4	10.9	0.456	0.030	14		11	26.9	15.3	9.4	0.350	0.023	−0.106	−0.007
Total		61	25.3	16.4	12.2	0.485	0.030	59		6.9	26.7	15.2	10.3	0.375	0.025	−0.110	−0.005

oculation. The weights of the mice as well as the weights of the tumors were taken after the individual had died. T/D is the result obtained by dividing the weight of the tumor by the survival time expressed in days $T/D//M$ was obtained by dividing the preceding quotient by the weight of the mouse. The last two columns were obtained by subtracting the corresponding values of T/D and $T/D//M$ obtained on the control and experimental animals—"E" representing Experimental and "C" representing Controls. These values are in nearly all cases lower for the experimental animals than they are for the controls. Whether this difference be significant or not is, however, very problematical.

[1] The data submitted in this paper were obtained while the investigator was resident at Bussey Institution, Harvard University, under appointment as Research Fellow of the Harvard School of Public Health.

[2] Strong, Leonell C., *Proc. Nat. Acad. Sci.*, **13**, No. 3, 1927.

THE INFLUENCE OF A POWER DAM IN MODIFYING CONDITIONS AFFECTING THE MIGRATION OF THE SALMON[1]

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The life history of the Pacific salmon includes a period of oceanic life followed by the migration of the adult fish to spawning grounds which are near the headwaters of coastal streams. The young hatched on these spawning grounds proceed down the same streams to the ocean but with this difference: that whereas the adults press their way vigorously and insistently upstream, taking no food in the journey and using every effort to overcome at once the obstacles met in the course of the ascent, the young fish move downstream very slowly spending considerable time feeding on the way and manifesting none of the apparent anxiety to complete the journey that the adults show.

The coastal streams which are utilized by the Pacific salmon come out of mountains near the shore. They are characterized by a rapid flow of water and considerable drop within a short distance. In the main, they are fed by glaciers or snow fields of the high mountains so that the water temperature is relatively low and the water flow in summer high. By virtue of these features these streams are peculiarly adapted to furnish a water supply for industrial purposes. Waterpower sites along the course of the streams have been taken up for industrial purposes and numerous power dams already built. The demand for water power makes it reasonably clear that a still larger number of high dams is likely to be constructed

within the near future. It is important both from the economic and from the scientific standpoint to ascertain in what way this modification of natural conditions affects the abundance and movements of the salmon.

The movements of the fish towards spawning grounds has been studied by a considerable number of investigators and certain of the influences which direct that movement have been traced to environmental conditions. The two influences which have been clearly recognized are the direction of the stream flow and the temperature of the water. It has long been known that once the salmon have entered the fresh water, it is impossible to make them turn back or to divert them from the established course. They move upstream with such determination that even when some insuperable obstacle is encountered they will not change their course but continue to utilize all their energies in the effort to surmount it. When in some way a natural barrier has been suddenly created in the course of such a stream or an artificial barricade erected, the fish remain at this point striving to find or make a way over or through the obstacle until they are totally exhausted. Efforts to persuade or compel them to retrace their course and select amother route have thus far proved unsuccessful.

The route followed by these salmon is often fairly complex. First of all they select only certain streams and never are found in adjacent apparently similar waters. As they mount the stream they pass by certain tributaries and choose another without apparent reason, making year after year in every case the same definite choice and rejecting opportunities for ascending to regions that appear to afford better opportunities for spawning than the points actually utilized. For many years I have been studying particularly the species known as the sockeye or red salmon. Some years ago[2] I was able to show that in certain instances at least this species which always spawns in a lake is moved at the junction of two streams to select the one having the lower temperature. Further studies on different streams have furnished confirmatory evidence of this condition and no instance has been found in which a sockeye salmon has at the fork of a stream taken the branch which showed a higher water temperature. Its progress upstream is determined by the impulse to buck the current and the route followed is fixed by its selection of the cooler water whenever a choice is offered.

The installation of power dams has evidently modified natural conditions considerably and indeed most strikingly in those two features which influence the migration of the salmon. This year in studying such a situation definitely evidence was secured to show that the changes in natural conditions are exercising on the migration of these fish an influence definite enough and extensive enough to bring about changes in the previous life cycle of the individual in these waters. These obser-

vations were made on the Baker River in the State of Washington.[3] Since I had studied the sockeye salmon on this stream during several previous summers and had extensive records of current and temperature conditions, it was possible to determine definitely the changes wrought by the erection of the power dam in question. This dam is located near the town of Concrete, just about the junction of the Baker with the Skagit river, and in a deep, narrow canyon. It is 260 ft. high. During the period of construction in 1925 and 1926, the movements of the salmon were necessarily greatly interfered with. At the present time the installation of a fish ladder and a cable hoist makes it possible to convey to the top of the dam those fish which can be persuaded to enter the pockets at the base of the fish ladder which is immediately adjacent to the tailrace and the power house and at the end of a rack built directly across the river.

Such a dam offers evidently an insuperable physical obstacle to the upstream movements of the fish. This fact which was recognized long ago furnishes the basis for provisions incorporated in certain state laws requiring the construction of fish ladders and of fish ways at dams. Such by-passes have been constructed in numerous instances and while in some cases they have worked with a certain degree of success, it is equally clear that in other cases they have failed to appeal to the fish and have not served the purpose of their installation. This may possibly be due to other less evident changes in the environment which nevertheless have been an influence in directing the activities of the fish. Some of these changes will be brought out by further consideration of the effects produced by the dam.

With the erection of the dam a considerable extent of the stream, often covering many miles, has been transformed from a rapid broken stream of relatively shallow water into a large deep reservoir or lake in which little or no current is discernible. Thus in Lake Shannon, formed by the Baker River dam at Concrete it was impossible to measure the current by means of a current meter and while a slight movement of the water could be detected it was nevertheless so insignificant that one may well question whether it would actually influence the fish. A rapidly moving stream has been replaced by a body of water at rest.

In the next place, the form of the installation has resulted in separating the lake into two very different regions. Water was flowing over the top of the dam only at occasional periods of excessive rainfall. Ordinarily the outflow took place through a tunnel opening at high water about 90 feet below the surface of the lake. Evidently the little current which actually did exist was limited to the region above that level whereas below the level of the intake canal the water was entirely devoid of current. The outflow after passing through the turbines was led away from the power house by a tailrace in which the water was in exceedingly rapid

movement. The tailrace joined the original stream only a short distance below the dam and some water was still flowing in the old channel between the foot of the dam and this point. Moreover, the water in that section was abundant and deep enough to afford opportunity for the ascent of salmon, though, excepting at periods of excessive rainfall, it did not approach in volume or strength the current passing through the tailrace.

Owing to its quiet surface and lack of movement, the water in the artificial lake formed by the dam was less highly oxygenated than in the rapid, rock-filled stream which previously occupied that region. Apparatus was not available last summer for testing exactly this condition but some crude tests that were made incline me to believe that under present conditions no serious modification in oxygen content has been brought about.

Temperature conditions in the newly formed lake have been most radically modified and differ from the original stream conditions in several respects. It was originally on this stream that I secured the evidence which led me to consider the difference in water temperature as the directive influence in bringing the sockeye to choose one branch of a stream rather than another. The Baker River is flowing nearly southeast at the point where it joins the Skagit while the latter stream at that point is flowing almost due west. In passing upstream the red salmon thus turns partly back on its course from the standpoint of compass directions when it leaves the Skagit which is the larger stream to enter the Baker River. But this shift is made by all the red salmon and no one of that species follows the Skagit upstream above the mouth of the Baker even though the upper regions of the Skagit and its tributaries furnish an abundance of apparently attractive lakes and spawning gravels. Temperatures taken at and near the junction of the two streams in former years showed that during the migrating season of the sockeye the Baker River is 1°F. or more lower in temperature than the Skagit. These observations were repeated this year and it was found that the same difference in the temperature existed but was even more marked in degree. Consequently, if the temperature of the water is the directive influence which determines the choice of route at stream junctions that influence has even been heightened by the construction of the power dam. However, one can forsee a temporary condition under which the reverse may be true. Should the power house be shut down the water from the surface of the lake flowing over the top of the dam would make the river water higher in temperature than before and, to judge from records obtained this year, higher than the Skagit at the junction, thus reversing the temperature relations of the streams. Somewhat similar conditions might be produced by excessive rainfall that would drive over the dam the warm surface water of the lake in large volume. Such conditions would naturally be infrequent and of

brief duration but the influence on the migrating sockeye would be most interesting to follow.

The next opportunity which the salmon has for making a choice between two streams is found only a short distance above the junction of the two rivers, namely, where the tailrace from the power house joins the Baker River. Here the stream from the tailrace offers usually a much larger volume of water and always water of much lower temperature. It was observed both last year and this year that the migrating salmon could not be kept out of the tailrace. Apparently they entered this on arrival from the lower waters and stayed in it fighting the current and striving to force an entrance into the turbines until thoroughly exhausted. They then drift out and congregate in the quieter waters of the old channel just above the junction with the tailrace. Here they could be observed in numbers, at different times making no more effort than sufficient to maintain themselves in position. Many of them bore evidence in wounds or fungused areas of struggle with the unfavorable conditions that they had encountered. I should mention that after such periods of rest some of these fish at least started up the old channel toward the dam and spent time and effort in fighting the rack which had been built to prevent access to the waters at the foot of the dam and which on the power house side of the river led to the base of the fish ladder mentioned before.

The temperature conditions in the lake are most highly modified and in two respects sufficiently radically to influence unfavorably the migration of the fish. These two features are, first, the greatly increased temperature of the surface water and, second, the very low temperature of the deep water.

The lake created by the dam is of considerable size being 8 to 10 miles in length and varying up to two miles in width. It is also rather deep, having a considerable area that is from one to two hundred feet in depth. A series of water temperatures was taken on three occasions and disclosed the general conditions which might be expected in a lake of that type and location. During the first week in August the surface water was about 66° to 68°F. in temperature. The thermocline was found at a depth of 12 to 15 ft. The observations showed a little greater variation in temperature on different occasions between this point and the level of the intake tunnel than is usual in such lakes. Below the level of the intake the temperature dropped very gradually. At 200 ft. it stood at this time about 41°F. The surface of the lake shows thus the existence of a considerable mass of water which is much warmer than would have been found previously in the river. On the other hand, there is a deep basin in the lake having a temperature at this time very considerably lower than the water flowing into the lake from the river above.

The salmon lifted over the dam sounded almost immediately but were

observed to return to the surface several times at short intervals before they finally disappeared. Only those which were fungused or otherwise in bad physical condition lingered in the surface waters.

Now the records for the summer of 1926 and for the past summer (1927) show that a considerable portion of the sockeyes lifted over the dam failed to reach Baker Lake. This is the normal spawning ground of this species; it is located 20 miles upstream from the dam. Thus 3578 fish were put over the dam in 1926 and only 2823 were caught at the lake. As a fish trap closes the entire channel where the river leaves the lake, the record is hardly open to question. During the past year 4158 fish were put over the dam and only 1328 reached the lake. Part of this difference is to be accounted for by the fact that some of the fish which were put over the dam were physically incapacitated and probably perished before they could have completed the upstream trip. On the other hand, it is clear that such an explanation is not adequate to account for the entire difference.

In other streams when red salmon have entered a lake with deep cold water, they rest and ripen in that water, continuing there until they rise to the shore zone to spawn. It is evident in that this deep cold area of Lake Shannon the fish would not be subject to a current stimulus such as under earlier conditions would have impelled them to continue the migration upstream and the temperature stimulus would tend to hold them in place since movement in any direction would bring them into warmer water. It is, of course, evident that such part of the fish as might continue along shore waters or through the upper water strata would find the river at the upper end of the lake and be impelled by the current stimulus to continue their ascent of the stream. These would then ultimately reach Baker Lake and the normal spawning grounds. Those fish which were trapped in the deep cold basin and ripened there would probably not find suitable spawning grounds or in the event that they did select some new place, the considerable variation in water levels in the new lake would imperil and very likely destroy the eggs in the course of the winter.

The downstream movement of the young fish has not yet been investigated. It is clear that their movement is seriously interfered with by the construction of the dam and the formation of the new lake. Some who have observed conditions at the dam maintain that the young fish go over the top with the excess water and reach the river at the foot of the dam successfully in spite of the drop of over 200 ft. It has also been stated that young fish go through the turbines without being killed. Both of these statements are deserving of careful study. It is certainly true that large numbers of what are probably red salmon remain in the lake and after the surface waters warm up, seek the deeper spots. They apparently retire below the thermocline when it is formed and spend the summer

in the deeper waters. There is no evidence to show whether they return to the surface after the fall overturn and continue their downstream migration or whether by virtue of the stop in their oceanward migration, they become a landlocked variety. Studies on these problems are in progress and it is believed that the situation will furnish valuable evidence regarding the effects of power dams on the migration of the sockeye salmon.

[1] Contribution from the Zoölogical Laboratory of the University of Illinois, No. 307.

[2] Ward, H. B., "Some of the Factors Controlling the Migration and Spawning of the Alaska Red Salmon," *Ecology*, **2**, 234–254, 2 figs., January, 1922.

[3] Research prosecuted under the auspices of the Department of Commerce, Bureau of Fisheries, Washington, D. C.

A NOTE ON THE TRIPLE SYSTEM, λ TAURI

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Communicated October 28, 1927

The star λ Tauri ($\alpha = 3^h55^m.1$, $\delta = 12°12'$) represents the well-known case of a triple stellar system. Spectroscopic observations by Dr. F. Schlesinger at the Allegheny Observatory[2] revealed the existence of the third body which revolves about the close eclipsing pair in a period (P_1) 34.60 days, the period of the latter being, according to Hellerich,[3] $P = 3.9529452 \pm 34$.

Dr. Schlesinger in his paper gave as approximate estimation of the dimensions of the two systems, the following:

Mass of the bright star	$m_0 = 2.5$
Mass of the eclipsing satellite	$m_1 = 1.0$
Mass of the distant satellite	$m_2 = 0.4$
Distance between the components of the eclipsing pair	$a = 11{,}200{,}000$ km.
Distance between the center of mass of the eclipsing pair and the third body	$a' = 50{,}000{,}000$ km.

These values were derived on some assumptions concerning the mass-ratio m_0/m_1 and the inclinations of both orbits.

Obviously enough, the elliptic elements of the two orbits, derived from spectroscopic observations, should be regarded as an approximation only, the accurate determination of the motion of the three bodies constituting a complicated problem of Celestial Mechanics.

The most conspicuous perturbation arising in the eclipsing pair under the influence of the third star would be the motion of the line of apsides,

the period of which, U (or its reciprocal $1/U$), can be expressed in terms of P, P_1 and $k \equiv m_0/(m_0 + m_1 + m_2)$ as follows:[4]

$$P/U = \frac{3}{4} k \left(\frac{P}{P_1}\right)^2 + \frac{225}{32} k^2 \left(\frac{P}{P_1}\right)^3 + \left(\frac{849}{32} + \frac{675}{128} k\right) k^2 \left(\frac{P}{P_1}\right)^4 + \ldots \quad (1)$$

The terms depending on the eccentricities of both orbits, and on their mutual inclination, enter the formula (1) only in higher approximations.

The expression (1) was deduced by considering the stars as material points. At the present time we have no adequate theory to estimate the influence of non-spherical shape of the stars. Tisserand[5] and, following him, Pannekoek[6] discussed the perturbations of this kind in the algol system. The shape of a component of a binary system was computed as the figure of equilibrium of incompressible fluid; afterward the star was treated exactly as a rigid body. As might be expected, the theoretical result disagreed with the observations; the computed advance of periastron was too great to explain a long-period term in the photometric formula for algol.

The homogeneous model of a star can probably not be used even as a first approximation. The main bulk of the mass of a star is concentrated about its center so that the tidal waves, which occur chiefly in the superficial layers, would not affect very much the gravitational potential. Furthermore, a star does not behave as a rigid body. The viscosity of stellar matter is probably small. Consequently, we may expect that a deformation of the star due to the external forces will partly replace the perturbation of the orbit. These two reasons lead us to believe that the influence of the non-spherical shape of the bodies cannot be large. Consequently, the formula (1) will be used alone as a close approximation.

Adopting the value of K from Dr. Schlesinger's estimate we find:

$$U = 10 \text{ years.}$$

J. Hellerich, having discussed all the available photometric observations covering the interval 1856–1921, pointed out that the periodic inequality in the photometric formula can be accounted for by the progressive motion of the periastron, if we assume $e = 0.020 \pm 0.007$ and $U = 14.8$ years.[3] The latter value according to our formula (1) corresponds to

$$k = 0.068.$$

This determination enables us to reduce the number of hypothetic assumptions which were used in order to estimate the dimensions of the system.

From the Allegheny results[2] we have

$$\frac{m_1^3 \sin^3 i_1}{(m_0 + m_1)^2} = 0.073 \pm 0.023, \; a_1 \sin i_1 = 3{,}050{,}000 \text{ km.} \pm 32{,}000 \text{ km.},$$

$$\frac{m_2^3 \sin^3 i_2}{(m_0 + m_1 + m_2)^2} = 0.0040 \pm 0.0006, \; a_2 \sin i_2 = 4{,}950{,}000 \text{ km.} \pm 250{,}000 \text{ km.}$$

where $a_1 = \frac{m_1}{m_0 + m_1} a$, $a_2 = ka'$. Using the value $\cos i_1 = 0.202$ from Professor J. Stebbins' photometric determination,[7] and assuming $i_2 = i_1$, we find:

$$m_0 = 10.2\odot, \qquad m_1 = 2.3\odot, \qquad m_2 = 0.9\odot,$$
$$a = 17{,}000{,}000 \text{ km.}, \qquad a' = 69{,}700{,}000 \text{ km.}$$

It has been remarked by Hellerich himself that there is no direct observational evidence of the motion of periastron. However, the data at present available are insufficient to permit any definite conclusion. Professor Stebbins in 1916–1920 could not detect any ellipticity of the orbit from his photometric observations, whereas the spectroscopic observations showed quite definitely the eccentricity, which reaches, in the case of the last Allegheny determination (1914), the value $e = 0.061 \pm 0.010$ with $\tilde{\omega} = 77^\circ.5 \pm 8^\circ.6$.

Discussing the observational material given in the Allegheny Publications, we notice at once that some of the normal places, formed to derive the spectroscopic elements of the orbit, lie within the time of eclipse. Consequently, the values of e and $\tilde{\omega}$ are liable to the rotational effect pointed out by Dr. Schlesinger,[8] and, therefore, should be ignored in the present discussions. To clear up the matter it is advisable to investigate the λ Tauri system in the way similar to that used by D. B. McLaughlin for algol.[9]

It can be shown that the dimensions of the λ Tauri system just obtained are fairly consistent with the Mass-Luminosity Relation. The ratio of the radius, R_0, of the bright star to the distance, a, between two eclipsing components is, according to Stebbins,[7]

$$R_0/a = 0.299.$$

Hence,
$$R_0 = 7.3 \text{ sun's radii.}$$

The comparison of λ Tauri with some stars of the same spectral type, *B3*, listed in the table XIX of Professor Eddington's book,[10] gives the absolute bolometric magnitude

$$m = -4.05 \text{ at } T_e = 17{,}000^\circ.$$

Hence, according to the Mass-Luminosity Relation,

$$m_0 = 10.2\odot$$

The discrepancy between this result and the value $m_0 = 10.2\odot$ given above can be easily accounted for by the uncertainty of the numerical data used in the computation. However, a complete accordance of the results can be reached by rejecting the assumption concerning the inclination, i_2, of the long-period orbit, and by carrying out the discussion of the problem in a different way.

Combining the expressions

$$m_1^3 = 0.0777(m_0 + m_1)^2 \quad , \quad a_1 = 4.48 \text{ sun's radii.}$$
$$m_1/(m_0 + m_1) = a_1/a \quad , \quad R_0 = 0.299a,$$

we obtain the equation

$$m_0 = 0.0434\, R_0^2(0.747\, R_0 - 1), \tag{2}$$

the units being the sun's mass and radius.

The radius R_0, when expressed as a function of the absolute bolometric magnitude m, is

$$R_0 = 1.138 \times 10^{-0.2m}, \tag{3}$$

or, if we reduce m to the "standard effective temperature" of 5200°,

$$R_0 = 1.259 \times 10.^{-0.2m} \tag{4}$$

The constants involved in these two expressions were derived—as before—by comparison of λ Tauri with u Herculis, Z Vulpeculæ, and U Coronæ.

Substituting the last expression for R_0, (4), in (2) we obtain the equation

$$\log_{10} m_0 = \log_{10}(1.365 \times 10^{-0.2m} - 1) - 0.4m - 0.839, \tag{5}$$

which is to be solved simultaneously with the Mass-Luminosity Relation. This is done graphically by drawing both curves on the same diagram. The point of intersection is: ($m = -3.20$, $\log_{10} m_0 = 1.139$). Consequently, the following elements of the eclipsing system are derived:

$$m_0 = 13.8\odot, \qquad m_1 = 2.8\odot, \qquad a = 26.7 \text{ sun's radii.}$$

The actual absolute bolometric magnitude of the bright star at $T_e = 17{,}000°$ is

$$m = -4.23.$$

The reduction of this value to the absolute visual magnitude gives

$$m_{vis.} = -2.89.$$

Assuming, from Stebbins' work, that the bright star contributes 0.796

of the total light, we find the absolute visual magnitude of the whole system to be

$$m_{vis.} = -3.14.$$

Hence the parallax of λ Tauri is

$$p = 0''.0042.$$

The present method of discussion can be applied to any eclipsing binary star in which the spectrum of only one component is observable.

Finally, adopting the previous value of $k = 0.068$ derived from perturbations in the eclipsing system, we get data concerning the long-period system:

$$m_2 = 1.2\odot, \qquad \sin i_2 = 0.9, \qquad a' = 120.0 \text{ sun's radii.}$$

In conclusion the results can be briefly summarized in the following table:

Period of the eclipsing system	P	3.4529	days
Period of the third body	P_1	34.60	days
Period of the motion of the periastron	U	14.8	years
Mass of the bright star	m_0	13.8⊙	
Mass of the eclipsing satellite	m_1	2.8⊙	
Mass of the distant satellite	m_2	0.9⊙	
Distance between the components of the eclipsing pair (sun's radius = 1)	a	26.7	
Distance between the center of the eclipsing pair and the third body	a'	120.0	
Radius of the bright stars	R_0	8.0	
Radius of the eclipsing satellite	R_1	6.0	
Density of the bright stars (sun's density = 1)	ρ_1	0.027	
Density of the eclipsing satellite	ρ_2	0.013	
Cosine of inclination of the eclipsing orbit	$\cos i_1$	0.202	
Sine of inclination of the long-period orbit	$\sin i_2$	0.9	
Absolute bolometric magnitude of the bright star	m	−4.23	
Absolute visual magnitude of the bright star	$m_{vis.}$	−2.89	
Absolute visual magnitude of the system		−3.14	
Apparent visual magnitude of λ Tauri at maximum		+3.75	
Parallax of λ Tauri	p	0''.0042	

[1] Fellow of International Education Board.

[2] *Publ. Allegheny Obs.*, **3**, Number 20, 1914.

[3] *Astron. Nachr.*, Number 216, pages 99–101.

[4] *Trans. Yale University Obs.*, **6**, Part III, 1927.

[5] *Comptes Rendus, Paris*, **120**, 125.

[6] Ant. Pannekoek, *Untersuchungen über den Lichtwechsel Algols*, Leiden, **1902.**

[7] *Astroph. J.*, **51**, pages 193–217, 1920.

[8] *Publ. Allegheny Obs.*, **3**, page 28.

[9] *Astroph. J.*, **60**, pages 22–31.

[10] A. S. Eddington, *The Internal Constitution of the Stars* (Chapters VI and VII), Cambridge, **1927.**

QUANTITATIVE ASPECTS OF TONIC IMMOBILITY IN VERTEBRATES

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Communicated November 15, 1927

I.—"Death feigning," otherwise "playing 'possum" or even "animal hypnosis," better termed *tonic immobility*,[1] is shown to a striking degree by a great variety of animals.[2,3,4,5] Certain forms from planarians to mammals may be caused to assume a state of rigid immobility by tactual stimulation of particular regions of the body or by other forms of manipulation. In the case of many of the higher vertebrates, including some mammals, the phenomenon has been said to be produced as a form of escape mechanism. Some writers have considered the "feint" to possess survival value and have spoken of it as an instinct of death feigning.[6]

Immobility periods in the case of most animals are characterized by certain physiological criteria, principal of which are a hypertonicity of the skeletal musculature and a non-reactiveness to environmental disturbances. The periods vary from a few seconds to several hours, depending on factors about to be discussed.

II.—In 1923 Crozier and Federighi[7] studied the effect of temperature on the durations of periods of tonic immobility of the isopod *Cylisticus convexus*. They had previously noted that when the animal was reimmobilized as soon as it had spontaneously recovered, the durations of the successive periods tended first to increase and then to decrease again in cyclical fashion, and that typically the cycle continued to repeat itself rhythmically as long as the process of reimmobilization was continued. They found that when the durations of the cycles were plotted against the temperature, according to the Arrhenius equation $\frac{k_2}{k_1} = e^{\frac{\mu}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$, good linear graphs were obtained. These functions gave two values of the constant μ, the critical thermal increment or *temperature characteristic*,[8,9] of the orders of magnitude typical for other chemically controlled biological processes.

It was also observed that the initial immobilizing stimulus in a series seems to start a chain of events which proceeds to a predetermined close even though this rhythm may be interrupted by premature forced "awakening" of the animal. In such cases the animal, if reimmobilized by the usual method† of gentle pressure on the thorax, continues to remain immobile until the time when it would have spontaneously recovered had the period not been interrupted. From these and other considerations,

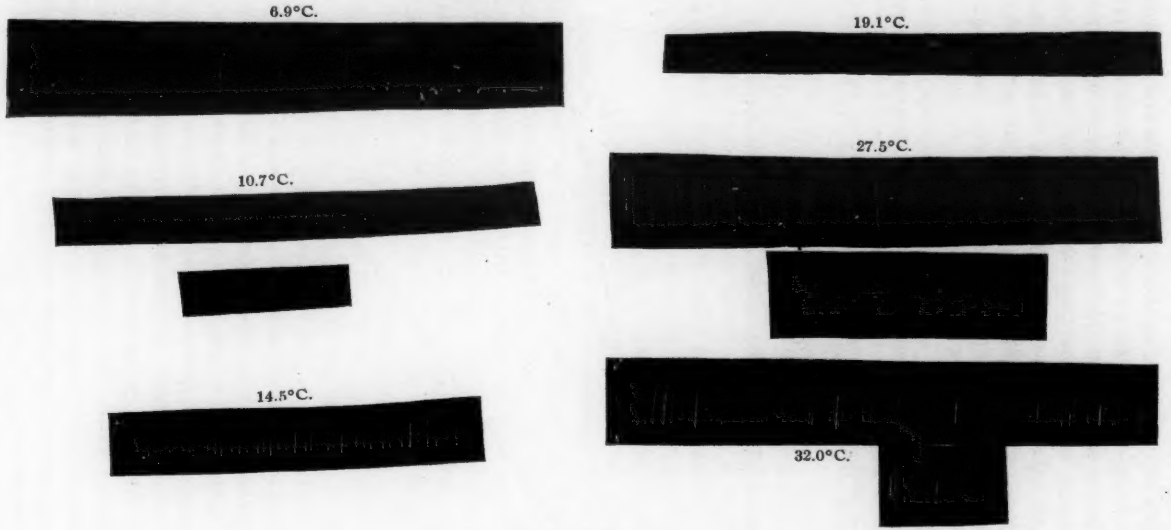

FIGURE 1

The vertical strokes in these records represent recoveries and reimmobilization of *Anolis*. The height of the vertical stroke has no significance, but the distance between the strokes is a measure of the time between spontaneous emergence from immobility. The three numbers at the left of the record from left to right, represent the number of the experiment, the number of the animal, in this case #7, and the temperature. It will be noted that the periods decrease with rise of temperature. At approximately 20°C. a new rhythm seems to be superposed on the original one, becoming most conspicuous at 27.5°C. This rhythm in turn shortens at higher temperatures. These are samples from approximately 200 records.

Crozier suggested that the duration of immobility might be considered as proportional to the amount of a substance which is an intermediate product in a chain of reactions and which is responsible for maintenance of tonic contraction of the skeletal muscles. This seemed to offer a means of approaching selective inhibition and integrative nervous action, and Crozier suggested that it might be extended to the consideration of the question of "synaptic resistance" in vertebrates.

III.—The problem has been pursued by the writer in the vertebrate field by an investigation of the mechanism of tonic immobility in the lizards *Anolis carolinensis* and *Phrynosoma strumosa*. An automatic mechanical

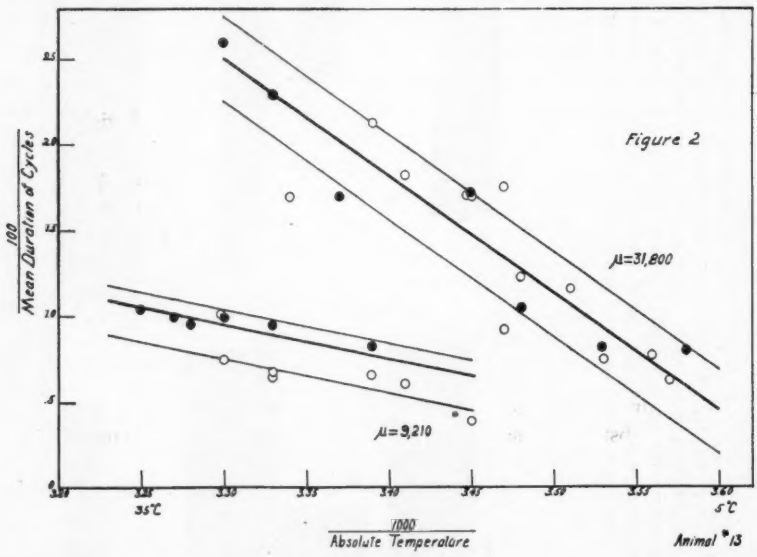

FIGURE 2

Typical plot of duration of rhythm, assumed proportional to amount of inhibitory autacoid, against temperature, according to the Arrhenius equation. The experiments were made at intervals covering over five months, with this individual. Heavy points are averages of at least five rhythms.

method was developed for controlling extraneous variables connected with arousal and excitation, of regulating the temperature and of recording periods of immobility, and a large number of kymograph records were obtained for analysis. For temperatures above that of the room the animals, with the necessary apparatus, were enclosed in an air thermostat. For lower temperatures a very accurate thermostat was employed consisting of a water bath cooled by coils in series with an SO_2 condenser pump which is operated by a relay in series with a mercury contact

switch immersed in the bath.[10] The chamber containing the animals was immersed in the water bath.

The mechanism of the tonic immobility was found to bear many points of basic similarity to that suggested by Crozier for arthropods, but certain striking differences became apparent (Fig. 1). The data of most of the records of durations of successive immobilizations were found to be fitted roughly by the Arrhenius equation (Fig. 2), giving evidence of chemical control of the durations of tonic immobility. Two characteristic μ values were obtained, one of which agrees closely with that obtained by Crozier with arthropods. However, in the case of *Anolis* there is no direct evidence for a set of catenary controlling reactions.‡ There seems rather to be two independent reactions. One of these is evident over the range 5 to 35°C., with μ = 31,000 ± calories. The other, which acts simultaneously but independently, i.e., is superposed, as it were, on the first rhythm (see Fig. 1), is effective between 5° and 20°C. and yields μ = 9000 ± calories. These critical thermal increments are associated with certain physiological oxidation processes, but the corresponding specific catalysts in this case are not known. The data were taken over a five-month period, and the final increments are averages from numerous experiments with six animals. This is sufficient to account for a rather wide latitude of variation found in the results, since in most determinations of thermal increments μ is calculated from data taken with one animal in a short period, usually not greater than 24 hours; this procedure was, of course, impossible in the present case.

In certain of the records there is evidence of an accumulation of the assumed substance responsible for maintenance of tonic contracture. A quantitative analysis of this effect shows that it may well be due to a limitation of diffusion of substance controlling the reaction, since, in terms of the basic assumptions, a diffusion curve describes the relationship in these cases. Moreover, the very small magnitude of the temperature effect in these cases is indicative of a physical rather than of a chemical process.

As in the case of arthropods, forced recovery of the animal in the midst of a rhythm has no effect on that rhythm or on subsequent rhythms. The reimmobilized animal continues the "feint" until the time for its normal recovery.

To account for these results, the existence of two autacoids is assumed, called α and β, which are conceived as decomposing to active chalones (inhibitory hormones) *A* and *B*. The latter substances appear to favor the passage of impulses from the tonic centers, at the same time inhibiting the higher centers. This mechanism is consistent with what little is known of the physiology of tonus in skeletal muscle and of the nature of inhibition.

Experiments are at present in progress to ascertain more definitely the nature of the autacoids in question. The injection of small amounts of adrenalin, in excess of a definite threshold amount, is found to prolong markedly the durations of tonic immobility in *Anolis*. The adrenalin is injected intraperitoneally (0.1 cc.), after diluting with Ringer's solution to concentrations varying with the experiments between 0.3:100,000 to 5.0:100,000. Controls injected with 0.1 cc. of Ringer's solution show no prolongation of immobility. The threshold values below which adrenalin seems to have no effect vary between 0.5 to 1.2 parts per 100,000, when 0.1 cc. of solution is injected. The effectiveness of the adrenalin in prolonging the periods seems, very roughly, to be proportional to the amount injected.

While it is possible that adrenalin may be one of the hypothetical autacoids, this is not certain in the light of other considerations. The injection of ergotamine, in the case of mammals, is known to paralyze the endings of the sympathetic nervous system. If it has a similar effect on lizards one would expect that if adrenalin is actually one of the autacoids that tonic immobility would be abolished. Exactly the opposite effect results. The duration of immobility is greatly prolonged by ergotamine. Since it is not certain how ergotamine affects the sympathetic nervous system of *Anolis*, this experiment is not in itself determinative. It is suggestive that larger injections of Ringer solution (0.2 cc.), which, as well as adrenalin and ergotamine, may be assumed to increase the blood pressure, prolong the duration of immobility. Amyl nitrite, on the other hand, which supposedly reduces blood pressure, abolishes tonic immobility altogether.

A detailed description of the foregoing experiments and their theoretical implications is soon to be published.

In addition to the question of the mechanism of tonic immobility, the condition itself offers opportunity for the study of tonus in skeletal muscle in the intact organism. The immobile animal resembles in many respects a decerebrate preparation. Certain aspects of deep reflexes are conspicuous, and experiments are in progress to measure the frequency of discharge of certain stretch reflexes of *Phrynosoma* as a function of the tension in the muscles.

The writer wishes to express his thanks to Professor W. J. Crozier, in whose Laboratory the experiments were done, for his many helpful suggestions throughout the course of the work.

Summary.—An analysis of tonic immobility in the lizard *Anolis carolinensis* leads to the assumption of the existence of two independent inhibitory hormones. These hormones are assumed to produce tonic immobility, when present above threshold concentrations, by inhibiting impulses from "higher nervous centers" and by allowing impulses to pass from the tonic centers to the muscles.

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† Tonic immobility seems to be essentially of an all-or-none character and its duration is quite independent of the intensity of the stimulus used in producing it. Arthropods seem to have fairly definite regions which, when stimulated, produce the phenomenon.[5] With vertebrates it may be produced readily by sudden turning into a dorsal position, by pressure on the thorax or in some cases by gentle stroking between the eyes.

‡ The temperature analysis was not carried out for *Phrynosoma*. It was found, qualitatively, that the general distribution of recovery periods resembles that with *Anolis*.

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SENSORI-MOTOR CONSEQUENCES OF PASSIVE ROTARY AND RECTILINEAR OSCILLATION OF THE BODY.

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Communicated October 22, 1927

Harmonic motion of the body in sine-wave rotary and rectilinear oscillation provided stimuli of constant form for an experimental exploration of the sensitivity of the vestibule and related human receptors. Two trained subjects (the writers), supported in a variety of experimentally appropriate ways on smoothly running platforms, were oscillated at various predetermined frequencies and amplitudes in rotary and rectilinear directions. They responded by compensatory hand movements to felt direction and amplitude of motion. The reactions can be described as an effort to keep the hands motionless in space by compensating for any perceived passive movement to which the body might be subjected. Graphic records gave directly comparable curves of platform movement, of compensatory reaction and, on occasion, of body sway.

The platforms[2] were oscillated in approximately true harmonic motion by an adjustable, constant speed motor acting on offsets from the platforms through a driving wheel and driving rod.

Rotary Oscillation.—Sensitivity of blindfolded subjects to rotary oscillation as measured by the percentages of right compensations is shown in fig-

ure 1. There is an almost rectilinear proportionality between average acceleration and manual compensation in the right direction from an average acceleration of the canals of *0.0026 cm. per second* to that of *0.1664 cm. per second*. The proportionality is apparent far below as well as above the region of the threshold or limen. The consistent differences of the two subjects suggest the probable utility of the method as a measure of the sensitivity of the semi-circular canals. The regularity of the results indicates the fundamental importance of acceleration in canal stimulation as compared with frequency.

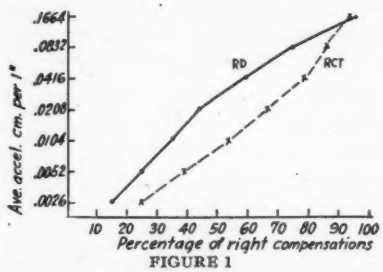

FIGURE 1

Relationship between the average acceleration and percentage of right compensations to rotary oscillation irrespective of the frequency of stimulation.

Rectilinear Oscillation.—Manual compensatory responses to the perception of rectilinear oscillation, represented by the curves in figure 2, show a systematic relationship between average acceleration and correct response similar to that in rotary oscillation. But the acceleration must be of a much higher order to evoke adequate compensations.

The relationship between frequency of rectilinear oscillation and correct response with average acceleration constant is shown by the curves in figure 3. Within the limits of our data the more rapid the oscillation at constant average accelerations the more adequate the response which, in our experiments, is a fair indication of the perception of oscillation.

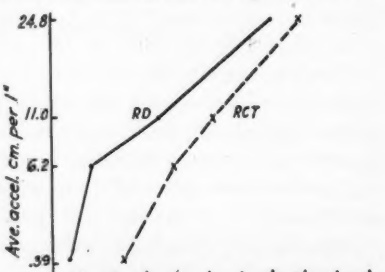

FIGURE 2

Relationship between the average acceleration and percentage of right compensations to rectilinear oscillation.

Comparative Sensitivity to Rotary and Rectilinear Oscillation.—Comparison of the sensitivity of the labyrinth to rotary and rectilinear oscillation, respectively, in terms of the average velocity of the bony labyrinth which evoked 50% of right responses shows that rotary oscillation is more effective by a ratio of 60 to 1 for subject RD and 81 to 1 for RCT. Analogous ratios in terms of average acceleration of the bony labyrinth are 382 to 1 for RD and 833 to 1 for RCT.

This difference in sensitivity of the labyrinth to rotary and rectilinear oscillation indicates either, that two separate receptors function in the perception of rotary and rectilinear oscillation, or that the same receptor has two different modes of excitability. The data given here and elsewhere[3] favor the former interpretation.

The Complication of Stimuli in Rectilinear Oscillation.—Experimental analysis of the rôles of various receptors shows that perception of motion resulting from passive rectilinear oscillation of a free standing subject involves not only rectilinear vestibular data and data of rotation caused by sway, but kinaesthetic data as well. In about 65% of these oscillations there is evidence that the kinaesthetic data determined the direction of compensation. Records of body sway showed that the bodily inertia produced a lag at the beginning of each half oscillation and that this lag was quickly overcome by an over-compensation, probably of reflex origin. This caused the body sway curves to be greater in amplitude than the platform curves. Manual compensation was evoked by the kinaesthetic data of reflex bodily over-compensation more often than by the perception of the real direction of motion and was consequently wrong in more than 50% of the instances of its appearance.

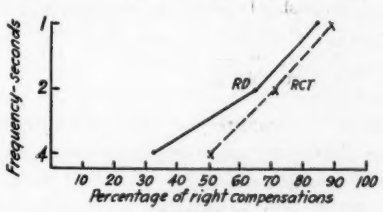

FIGURE 3

Relationship between frequency of stimulation and percentage of right compensations with constant average acceleration.

Visual cues to the direction of movement in either rotary or rectilinear directions proved to be of a very much higher order of adequacy than either the kinaesthetic or the vestibular. Accelerations represented in figure 1 never failed to evoke a correct compensation when the subject could see the environment. When the subject was free to move, the kinaesthetic receptors for rectilinear oscillation proved to be at least ten times as sensitive at the vestibular.

Summary.—1. In both passive rotary and rectilinear oscillation of blindfolded subjects, there is almost rectilinear proportionality between average acceleration and compensation over a wide range on either side of the threshold.

2. Within the limits of our data the higher the frequency of rectilinear oscillation the more adequate the perception of motion and its direction.

3. The vestibular receptors for rotary oscillation are more sensitive than those for rectilinear oscillation.

4. Compensation is most adequate when evoked by stimulation of the visual receptors. In the order of their sensitivity the receptors for rectilinear oscillation are visual, kinaesthetic and vestibular.

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FOSTER PARENT-FOSTER CHILD COMPARISONS AS EVIDENCE UPON THE NATURE-NURTURE PROBLEM[1]

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The question of nature *versus* nurture has existed as a recognized problem since Galton's publication in 1869 of "Hereditary Genius."[2] Following this pioneer work many other studies on family resemblance in psychical traits employing more and more refined techniques, and culminating in the mental test investigations of the present day, have appeared. The latter have shown beyond doubt that blood relatives resemble each other to about the same degree in mental traits as in physical traits known to be hereditary.

Since the results of such experiments—the consistent decrease in familial correlation coefficients as more and more remote degrees of relationship are considered, the high incidence of genius in certain human strains and of feeble-mindedness in others, the stronger resemblance found between twins than between ordinary siblings—might conceivably be due either to hereditary or to environmental forces or to both at once, all these data have been seized upon both by the hereditarians and the environmentalists and interpreted as favorable to the point of view of each school.

It was to untangle the factors of home environment and heredity that correspondences between the mental status of foster parents and foster children were compared with those between true parents and true children. Since heredity was ruled out in the foster group (and since resemblance due to selective adoption was also largely ruled out by our criteria for inclusion of cases), it was possible to determine how much of ordinary family resemblance is due to the influence of environment.

The data consist of Stanford-Binet intelligence test records upon 200 sets of foster parents and foster children, and upon 100 sets of true parents and true children, together with environmental data upon the cultural, educational and material aspects of the homes.

Our foster children were placed in their foster homes during the first year of life (the average age of placement being only three months).

The children were all between the ages of five and fourteen inclusive when tested. Not enough was known regarding their true parentage to make effective selective placement possible. Only children placed with white, English-speaking foster parents were included. In selecting our control group of true parents and true children, the foster children were closely matched for age and sex, and the foster parents for intelligence and occupational status. This matching made it possible to use the control group as a check upon the validity of the methods employed impartially with both groups.

The results are best presented as correlation coefficients between the intelligence quotients of the children and certain environmental factors. Raw correlations and correlations corrected for attenuation (i.e., corrected for the unreliability of our test scores and environmental measures) are tabled below.

CORRELATION OF CHILD'S I. Q. WITH:	FOSTER GROUP r	FOSTER GROUP P.E.	CONTROL GROUP r	CONTROL GROUP P.E.
Father's (or foster father's) mental age				
Raw	0.07	0.05	0.45	0.05
Corrected for attenuation	0.09	0.06	0.55	0.06
Mother's (or foster mother's) mental age				
Raw	0.19	0.05	0.46	0.05
Corrected for attenuation	0.23	0.06	0.57	0.06
Mid-parent mental age				
Raw	0.20	0.05	0.52	0.05
Culture index of home				
Raw	0.25	0.05	0.44	0.05
Corrected for attenuation	0.29	0.06	0.49	0.06
Nine environmental variables (multiple r)				
Raw	0.35	0.05	0.53	0.05
Corrected for attenuation	0.42		0.61	

The culture index of the home was a combined rating of the parental education, speech habits and interests, of the size and quality of the home library, and of the home furnishings. The nine variables used in the multiple correlation were: father's mental age, mother's mental age, father's vocabulary, mother's vocabulary, father's schooling, mother's schooling, material status of home, cultural status of home and family income. Most of these variables, of course, overlap to some extent with one another. Indeed, it was found that the "best" four of the nine variables used in the multiple correlations were practically as efficient as all nine variables in estimating the criterion (child's I. Q.). The conclusion was thereby drawn that the multiple correlation reported for the foster group in the above table represents close to the maximum contribution of measurable factors in home environment to differences in children's intelligence. (In this connection see Pearson's discussion of the interpretation of mul-

tiple correlations employing a large number of variables.[3]) Following Fisher[4] the square of the multiple correlation was taken as the total percentage contribution of measurable home environment to the variance of intelligence. The contribution of home environment to intelligence is thus the square of 0.42, or 17 per cent.

The correlations in the control group serve as a check upon the experiment since they agree well with similar correlations found by previous investigators, and demonstrate that the methods here employed are adapted to revealing family resemblance when it exists.

Although the multiple correlation for this group shows that only 0.61[2] or 37 per cent of the variance of children's I. Q.'s is accounted for by reference to environment and the intelligence of their true parents combined, it is probable that the major share of the residual variance is due to congenital endowment, since in known modes of hereditary transmission the influence of heredity is always far stronger than parental correlations alone would indicate. This is necessarily the case, inasmuch as only half the chromosomes of each parent are passed on to the offspring, so that the parental deviation for any trait in question is determined by a number of factors other than the ones transmitted to the child.

Using the value, 15, which is the standard deviation of children's I. Q.'s, and 0.42, the corrected multiple correlation of environment with the I. Q.'s of foster children, the conclusion is also reached that measurable environment one standard deviation above or below the mean of the general population does not shift the I. Q. by more than 6 to 9 points above or below the value it would have had under normal environmental conditions. In other words, nearly 70 per cent of school children probably have an actual I. Q. of within 6 to 9 points of that represented by their innate intelligence. Taking the best environment to be three standard deviations above the general mean, the maximum contribution of home environment to intelligence is apparently about 20 I. Q. points; and, conversely, the least cultured, least stimulating kind of American home environment may depress the I. Q. as much as 20 I. Q. points. But situations as extreme as either of these probably occur only once or twice in a thousand times in American communities with school facilities.

A fuller account of this study will appear shortly in the 1928 Yearbook of the National Society for the Study of Education.

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THE THERMAL AND PHOTOCHEMICAL DECOMPOSITION OF AZO COMPOUNDS AND THE PROBLEM OF REACTION RATES

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The discovery that the decomposition of azomethane[2] (CH_3NNCH_3) is homogeneous and unimolecular suggested the probability that all of the azo compounds will decompose in like manner. This type of compound decomposes at lower temperatures than aldehydes, ketones and ethers, some of which decompose unimolecularly,[3,4,5,6] but at such high temperatures that the first products formed are not the most stable ones. Consequently, the rate constants of both azomethane and azoisopropane are more accurate. They have also been determined to much lower pressures. The data on these two reactions now give an opportunity to test various theories of reaction velocity.

Azomethane[2] was found to decompose almost entirely by the equation $CH_3NNCH_3 = C_2H_6 + N_2$. At high pressures the unimolecular rate constant was independent of the initial pressure of the gas. From the change of rate with temperature the heat of activation was found to be 51,200 cal. per mol. At 4 cm. initial pressure the rate constant was appreciably lower and it became steadily lower as the initial pressure was decreased until at 0.025 cm. the rate had dropped to 22% of the high-pressure value when the temperature was 290°C. and to 10% for 330°C.[7] Experiments just completed with azoisopropane show that *the rate constant does not become lower even at a pressure of 0.025 cm.*

Azoisopropane is a light yellow liquid boiling at 88.5°C. and was prepared by the method of Lochte, Noyes and Bailey.[8] Rate constants were determined at temperatures from 250°C. to 290°C. and initial pressures from 0.025 cm. to 4.60 cm. The method used was similar to that used with azomethane at low pressures,[7] except that it was necessary to surround the McLeod gage with an air bath at a temperature high enough to prevent condensation of the gas in the capillary of the gage. An increase of five-fold in the surface by means of small pyrex tubing made no difference in the rate even at the lowest pressures. The heat of activation is found to be 40,900 cal. per mol. ± 500 cal. The rate constant is given by the equation $K = 5.6 \times 10^{13} \times e^{-\frac{40900}{RT}}$. It has recently been shown by Hibben[9] that the rate constant of the decomposition of nitrogen pentoxide is constant to very low pressures.

We have now at least these two unimolecular reactions which show no

reduction in the rate at the lowest pressures for which accurate results are possible. In addition we have a number of reactions[3,4,5,6,7] which are strictly first order only at high pressures, and at least two which have not been studied at low pressures.[10,11] These may be taken as strong evidence in favor of a collision theory of activation since the argument of Perrin which necessitated a radiation theory of chemical reaction if the rate was independent of pressure even at infinitely low pressures is no longer valid. Furthermore, the radiation theory has been disproved by Lewis and Mayer[12] at least in the case of the racemization of pinene. Earlier attempts to provide activated molecules by collisions at a rate sufficient to explain the experimentally observed rates failed because they disregarded the internal energy as a source of activation energy. This was first pointed out by Lewis and Smith[13] and was extended by Hinshelwood and Lindemann[14] and by Christiansen.[15] Later, Rice and the author[16] developed a more complete equation for the same theory (theory 1) which showed the way in which the rate would fall off with decreasing pressure. This theory required only that the total energy of the molecule should equal or exceed a minimum energy and that the chance that an activated molecule should react was independent of the amount of energy in excess of this minimum. A second theory (theory 2) required that a certain minimum of energy should be located in one degree of freedom of the molecule to cause its reaction. The energy within the molecule could be redistributed among the various degrees of freedom between collisions. In this theory the chance of reaction depends upon the excess of energy in the entire molecule beyond the minimum required in a particular degree of freedom.

In a second theoretical paper Rice and the author[17] have applied these theories to the decomposition of azomethane. Using theory 2 and assigning 25 degrees of freedom we were able to show that the theoretical curve of $\log K/K_\infty$ against $\log p$ coincided with the experimental curve very well. Here K is the rate constant (which becomes lower as the pressure is lowered), K_∞ is the rate constant at high pressures and p is the pressure. It should be noted that theory 1 could not fit the data at all over the 3000 fold pressure range. Furthermore, theory 2 requires that $\log K/K_\infty$ decrease more rapidly with decreasing pressure at the higher temperature which is experimentally the case, while theory 1 has practically no such temperature coefficient. The theory of Fowler and Rideal[18] was used by Bernard Lewis[19] to explain the decomposition of azomethane before the data on azomethane at low pressures was published. This theory has not been developed so far as to show the manner in which the rate will change at low pressures. It requires fewer degrees of freedom but has the disadvantages pointed out by Tolman, Yost and Dickinson.[20]

Two new experimental results will now be considered in relation to

theory 2. First, the decomposition of azoisopropane can only be explained by allowing at least 50 degrees of freedom. A similar number of degrees of freedom is required on theory 1. The large size of the molecule, however, does not make this impossible. Twenty-five degrees of freedom were required for azomethane[17] and this was not inconsistent with the specific heats of molecules of similar size and structure. Azomethane has ten atoms and ten chemical bonds while azoisopropane has twenty-two atoms and twenty-two chemical bonds.

Second, the photochemical decomposition of azomethane was carried out by the author, with the result *that approximately two molecules were decomposed by each quantum of light absorbed over a wide range of pressures.* The quantitative absorption curve for azomethane gas was determined from 4060 to 2370 Å by the method of Ramsperger and Porter.[21] There was an absorption maximum at 3390 Å with increasing absorption again toward the far ultra-violet. No evidence of a band spectrum was obtained. Nearly monochromatic light was obtained by use of a filter of phenosaphranine solution in a cell containing cobalt glass windows. The three lines, 3650, 3654, and 3663 Å and a small amount of deep red and ultra-red was practically the only light transmitted by this filter. The total energy falling on the reaction cell was determined by means of a thermopile and the fraction of light absorbed was calculated from the absorption coefficient. The rate of decomposition was followed by pressure measurements. Four experiments were made at pressures from 0.1207 cm. to 25.74 cm. The temperature coefficient was found to be very nearly zero.

The nature of the photochemically activated molecule may not be similar to molecules activated thermally with the same total amount of energy and it is not certain that the mechanism of the two types of reactions is the same. We may, however, determine whether the result of this photochemical experiment is the result expected if the mechanism of the two types of reactions is alike. The equations of theory 2 permit us to calculate the ratio of activated molecules of given energy which react to those which are deactivated. This ratio is b_ϵ/aN where b_ϵ and a are given by equations (18) and (3) of reference 16 and N is the number of molecules per cc. If we take for the energy ϵ that of the quantum (77,800 cal. per mol.) plus the energy of an average molecule (roughly 5000 cal. per mol.), $\epsilon_0 = 50{,}600$, $n = 25$, $T = 563$, $K = 1.38 \times 10^{-4}$, $s = 6 \times 10^{-8}$ and N corresponding to a pressure of 25 cm. we find for this ratio 47/1 and the ratio is, of course, much greater at low pressures. That is, reaction is practically certain at pressures of 25 cm. or less. If we assume deactivation at collision (which was assumed in the derivation of theory 2) this result requires that a molecule of this energy must react in less than 2×10^{-10} sec. which is the time between collisions at 25 cm. pressure. But this would also be required by the constant photochemical efficiency

over the large pressure range of these experiments. Since there was not a long reaction chain in the photochemical decomposition it seems likely that the thermal reaction which is due largely to molecules of considerable less energy is not a chain reaction. The quantum efficiency of the dissociation of HI was found by Bernard Lewis[22] to be unchanged at 0.1 mm. and here, too, the molecule must dissociate before collision at high pressures.

It is to be noted that any of these first order reactions will become second order on either theory at sufficiently low pressures but that these pressures are, in the case of very large molecules, lower than can be attained. It was noted by Hinshelwood that all bimolecular reactions were those of simple diatomic or triatomic molecules while all unimolecular reactions were of more complex molecules. Analogously, azoisopropane is strictly unimolecular while the simpler azomethane deviates considerably at low pressures. We may expect that the compound CH_3NNH (methyl diimide) would only be unimolecular at very high pressures and bimolecular at the lowest possible pressures. An attempt will be made to prepare this unknown compound. The compound N_3H will also be tried but will probably be found to decompose bimolecularly if the decomposition is homogeneous.

Finally it is the opinion of the writer that not sufficient is known about the transfer of energy between molecules at collision or between internal degrees of freedom to account very accurately for all homogeneous reactions. The fundamental assumptions of all of the collision theories differ on these questions. What little evidence we have indicates some very specific effects as, for example, the experiments of Stuart[23] which show that the quenching of the resonance fluorescence of mercury differs very widely for different gases being greatest for H_2 and least for He. The experiments of Hinshelwood show that of a number of inert gases only hydrogen can maintain the high pressure rate constant of a "quasi" unimolecular reaction at low pressures. Nevertheless, it seems probable that the more complicated the molecule the more likely it is that the assumptions of statistical mechanics and of statistical equilibrium at collision will be fulfilled, and that the fewer quantum states of simpler molecules may introduce special restrictions of energy transfer. In agreement with this view it was found with azomethane that at low pressures the first order rate constant was reasonably constant from one time interval to the next in any given experiment, indicating that the products of the reaction had about the same influence in maintaining the rate as their equivalent of azomethane would. The products from a simple molecule decomposing bimolecularly do not contribute toward activation else the reaction would not be second order throughout its course.

The complete results of the photochemical decomposition of azomethane and of the decomposition of azoisopropane will be published elsewhere.

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